

## CHAPTER 2

### Energy Conversion

#### Introduction

A water flow from an upper level to a lower level represents a hydraulic power potential. This power flow can be utilised in a water power plant by conversion to mechanical power on the shafts of turbines. However, some fractions of the power potential are lost partly in the plant's conduits and partly in the turbines.

In this chapter a brief description is given of the power conversion in the turbines that are common in hydro power stations.

#### 2.1 Fundamentals and definitions

##### *Specific energy*

The specific energy of a hydro power plant is the quantity of potential and kinetic energy which 1 kilogram of the water delivers when passing through the plant from an upper to a lower reservoir. The expression of the specific energy is Nm/kg or J/kg and is designated as  $[m^2/s^2]$ .

In a hydro power plant as outlined on Fig. 2.1, the difference between the level of the upper reservoir  $z_{res}$  and the level of the tail water  $z_{tw}$  is defined as the *gross head*

$$H_{gr} = z_{res} - z_{tw} \quad (2.1)$$

The corresponding *gross specific hydraulic energy*

$$E_{gr} = gH_{gr} \quad (2.2)$$

where  $g$  is the acceleration of gravity.

When a water discharge  $Q$  [ $m^3/s$ ] passes through the plant, the delivered power is

$$P_{gr} = \rho Q g H_{gr} \quad (2.3)$$

where  $P_{gr}$  is the *gross power* of the plant

$\rho$  is the density of the water

$Q$  is the discharge

To look further on the hydropower system in Fig. 2.1 the specific hydraulic energy between the Sections (1) and (3) is available for the turbine. This specific energy is defined as *net specific energy* and is expressed by

$$E_n = gH_n \quad (2.4)$$

$$\text{and the net head of the turbine } H_n = \frac{E_n}{g} \quad (2.5)$$

As shown on Fig. 2.1 there are two ways of expressing the evaluation of the net head. The one way

$$H_n = h_p + c^2/2g$$

And the other way

$$H_n = H_{gr} - \frac{E_L}{g} = H_{gr} - H_L$$

where  $h_p$  is the piezometric head above tailwater level measured in section (1),  $c^2/2g$  is the velocity head in section (1) and  $E_L/g$  is specific hydraulic energy loss between reservoir and section (1) converted to head loss  $H_L$ .

**Note.**

Some comments to the specific energy definitions according to Equations (2.2) and (2.4) should be mentioned. For efficiency tests of hydro turbines a relatively high exactness of the determination of the specific energy is required. Therefore an international standard exists for the measurements and evaluations of

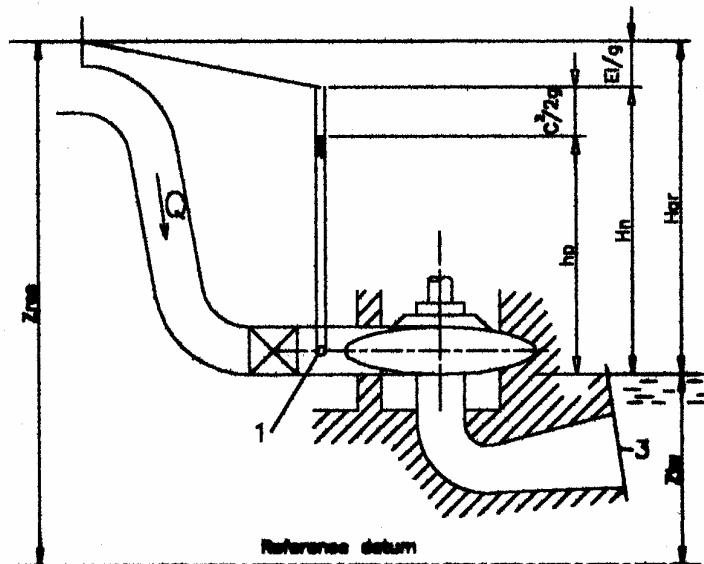


Fig. 2.1 Hydro power plant. Definition of gross head  $H_{gr}$  and net head  $H_n$

such tests. The name of it is **International Standard IEC 41**.

In addition to the specifications of relevant levels this standard take into account the influences of: Compressibility and temperature effects of the water; the weight of the air column difference between the reservoir and the tail water; the difference of specific kinetic energy between defined sections of the system and at last that the acceleration of gravity depends on the altitude and latitude.

The specific energy expressed by putting  $H_{gr}$  corresponding to Fig. 2.1, in Equation (2.2) and  $H_n$  in Equation (2.4) respectively, is consequently approximations according to this standard. However, the mentioned influences are relatively small, i.e., totally of the order 1% in extreme cases. This means that these influences are essentially smaller than the tolerance accuracy of the hydraulic dimensioning of the turbomachines. Therefore the hydraulic considerations of calculation and design in the following sections are based on constant values of the acceleration of gravity and the density of water, no influence of temperature and the weight of the air column.

## 2.2 Transforming hydraulic energy into mechanical energy

### 2.2.1 General considerations

#### *Ordinary turbines.*

The discharge and the net head for turbines differ in wide ranges from one power plant to another. This indicates that not only different types of turbines but also a very large register of sizes of turbines are needed.

The ordinary turbine types in water power plants are distinguished in two main groups:

- impulse turbines
- reaction turbines

This distinction is based on the difference between the two cases of energy conversion in these turbines. Briefly these two ways of energy conversion may be pronounced as follows.

- Basically the flow energy to the *impulse turbines* is completely converted to kinetic energy before transformation in the runner. This means that the flow passes the runner buckets with no pressure difference between inlet and outlet. Therefore only the impulse forces being transferred by the direction changes of the flow velocity vectors when passing the buckets create the energy converted to mechanical energy on the turbine shaft. The flow enters the runner at nearly atmospheric pressure in the form of one or more jets regularly spaced around the rim of the runners. This means that each jet hits momentarily only a fraction or part of the circumference of the runner. For that reason the impulse turbines are also denoted *partial turbines*.
- In the *reaction turbines* two effects cause the energy transfer from the flow to mechanical energy on the turbine shaft. Firstly it follows from a drop in pressure from inlet to outlet of the runner. This is denoted the *reaction part* of the energy conversion. Secondly changes in the directions of the velocity vectors of the flow through the canals between the runner blades transfer impulse forces. This is denoted the *impulse part* of the energy conversion. The pressure drop from inlet to outlet of the runners is obtained because the runners are completely filled with water. Therefore this group of turbines also have been denoted as *full turbines*.

The most commonly used turbines today are:

- impulse type: *Pelton turbines*
- reaction type: *Francis turbines*  
*Kaplan turbines*  
*Bulb turbines*

In the following sections the hydraulic energy transfer in Pelton turbines as well as the reaction turbines is described in more detail <sup>/1/,/3/,/4/</sup>.

### 2.2.2 Impulse turbine - Pelton

A section through a Pelton runner is shown in Fig. 2.2. The water jet from the nozzle hit the buckets that are spaced equidistant around the runner disc. In the left lower corner of the figure a look into a bucket in the same direction as the incoming jet is shown. To the right of the figure a section through two neighbouring buckets are shown for the indicated section line A - A. In this section is shown how the jet flow is split symmetrically at the bucket edge and passing over the bucket. The jet deflection is nearly up to 180°, but limited to a little smaller angle because the leaving jets have to run clear of the subsequent bucket behind.

For a net head  $H_n$  the theoretical velocity of the water jet out of the nozzle is found according to Bernoulli's equation

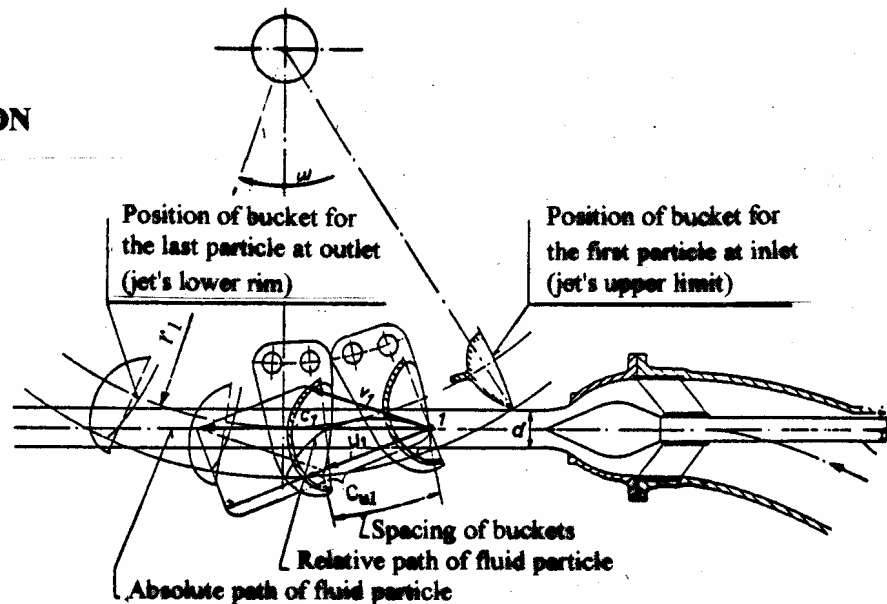
$$c_1 = \sqrt{2gH_n} \quad (2.6)$$

However, an energy loss occurs in the nozzle. This influence is corrected for by a friction coefficient  $\varphi$ , and the velocity then becomes

$$c_1 = \phi \sqrt{2gH_n} \quad (2.7)$$

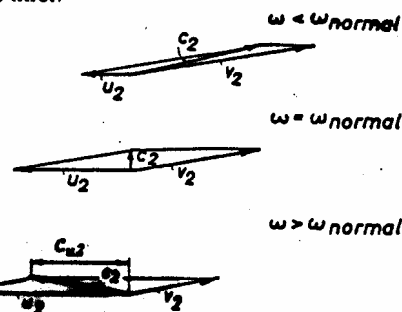
A value of the friction coefficient based on experience may be  $\phi = 0.98$ .

### RADIAL SECTION

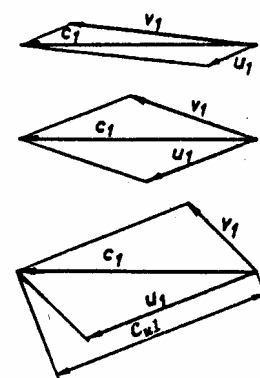


### VELOCITY DIAGRAMS

at inlet:



at outlet:



### AXIAL SECTION

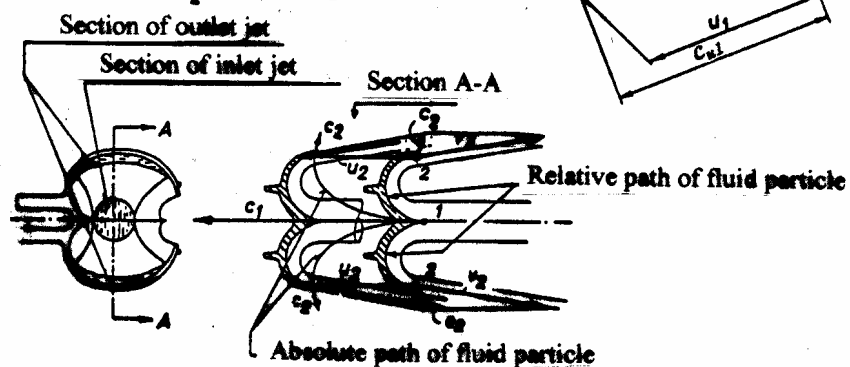


Fig. 2.2 The water jet flow into a Pelton runner. Velocity diagrams at inlet and outlet of the buckets /3/

The runner is assumed to rotate with a constant angular speed  $\omega$ .

One water particle is considered, for example the particle in the centre of the jet just at the splitting edge of the bucket and marked as position (1) on the Fig. 2.2. This bucket is drawn for a position corresponding just to the moment that the full jet is entering the bucket. The *absolute*

velocity  $c_1$  is known from Equation (2.7). The *peripheral* velocity of the runner  $u_1 = r_1\omega$  corresponds to radius  $r_1$  in position (1). The direction of this velocity vector is the same as the tangent in position (1) of the circle with radius  $r_1$ .

When the absolute velocity  $c_1$  of the water particle and the peripheral velocity  $u_1$  in position (1) are known, the velocity  $v_1$  of the water particle *relative* to the bucket in position (1) can be found. The absolute velocity  $c_1$  is the *geometric sum* of the peripheral velocity  $u_1$  and the relative velocity  $v_1$ . Therefore one has to draw the three velocity vectors from point (1) so that  $c_1$  is the diagonal in the parallelogram with  $u_1$  and  $v_1$  as sides. This parallelogram is called the *velocity diagram* of the water particle at the inlet of the bucket.

The water particle moves over the bucket and changes its direction gradually until it leaves the bucket at position (2) as shown in section A - A on Fig. 2.2. During this movement the particle transfers its impulse force corresponding to the change from the direction of the relative velocity vector  $v_1$  to the relative velocity vector  $v_2$ . The magnitude of  $v_2$  depends on energy losses during the passage of the bucket. These losses can be expressed as  $\zeta_2 \frac{v_2^2}{2}$  where  $\zeta_2$  is defined as loss coefficient. From experience an estimated value of  $\zeta_2 = 0.06$ . The relation between  $v_1$  and  $v_2$  is found according to Bernoulli's equation:

$$h_1 + \frac{v_1^2}{2g} = h_2 + \frac{v_2^2}{2g} + \zeta_2 \frac{v_2^2}{2g} \quad (2.8)$$

In this case  $h_1 = h_2$ , and therefore

$$(1 + \zeta_2) \frac{v_2^2}{2g} = \frac{v_1^2}{2g} \quad (2.9)$$

and

$$v_2 = \frac{v_1}{\sqrt{1 + \zeta_2}} \quad (2.10)$$

The magnitude of velocity  $v_2$  is nearly as great as  $v_1$  and has a direction as shown in position (2), section A - A on Fig. 2.2. The peripheral velocity  $u_2$  is assumed the same as  $u_1$  because in an approximate approach it is supposed that the water particle enters and leaves the runner bucket at the same radius of the runner. Therefore the velocity diagram may be drawn from position (2) with  $u_2$  and  $v_2$  as the parallelogram sides and the absolute velocity  $c_2$  as the diagonal. This velocity diagram shows that the magnitude of  $c_2$  is much smaller than of  $c_1$ , which is just the main intention to obtain, because  $c_2^2/2$  is a direct measure for the loss at outlet of the turbine.

The passage of the water particle from position (1) to position (2) lasts a certain time interval, and during this time the runner rotates a corresponding angle. If the corresponding positions of the bucket and the absolute flow path under this passage over the bucket are drawn, one gets an absolute flow path which ends up with the velocity vector  $c_2$  as shown in section A - A on Fig. 2.2. The absolute velocity vector is therefore tangent to this path all the way of the passage.

The description of the movement over the bucket of one water particle is valid also for all the other particles of the full jet. These particles will however trace different paths, but even so, in practise it is assumed that the impulse force and the corresponding torque transfer to the runner is the same from all water particles in the jet.

When the runner is rotating and the bucket starts splitting the jet, it gradually takes up more and more of the full jet area until it cuts the jet completely. But immediately after this cut the

succeeding bucket starts splitting the jet and repeats the same behaviour as the preceding bucket. In the time interval between the first water particle enters and the last one leaves a bucket at a distance as indicated on Fig. 2.2, is covered.

The total deflection of the jet may be made greater for a relatively large spacing between the buckets compared with a smaller one. Therefore the spacing between the buckets is made as big as possible, but not larger than to secure that all water particles will hit the bucket.

With the same torque transfer to the runner from all water particles in the jet, the velocity diagrams for the inlet and outlet respectively for one of the water particles is valid for all the water particles of the jet. The general expression for the transferred power  $P_R$  is therefore

$$P_R = \rho Q(u_1 c_{u1} - u_2 c_{u2}) \quad (2.11)$$

where  $Q$  is discharge

$u_1$  is the peripheral velocity of the runner where the jet hit the bucket

$u_2$  is the peripheral velocity of the runner at jet outlet of the runner

$c_{u1}$  is the component of the absolute velocity  $c_1$  in the direction of  $u_1$

$c_{u2}$  is the component of the absolute velocity  $c_2$  in the direction of  $u_2$

As previously mentioned the peripheral velocity  $u_1 = u_2$  for the Pelton turbine. Moreover it is suggested to insert the peripheral velocity corresponding to the runner diameter to which the centre line of the jet is tangent. The power equation then becomes:

$$P_R = \rho Q u_1 (c_{u1} - c_{u2}) \quad (2.12)$$

How this power varies if the rotational speed is changed, is interpreted as follows. As basis is ascertained that the absolute velocity  $c_1$  is constant and therefore  $c_{u1}$  is constant. The discharge  $Q$  is constant while the angular velocity  $\omega$  is varied. The rotating speed  $u_1 = r_1 \omega$ . From Fig. 2.2 it is found that  $c_{u2}$  varies when  $\omega$  is varied. In the case  $u_1 = 0$ , i.e., the runner is at standstill, the power  $P_R = 0$  and  $c_{u2} \approx -c_{u1}$ . When the runner is rotating, the velocity component  $c_{u2}$  decreases as the rotational speed increases and it approximates to zero when  $u_1$  increases towards  $c_1/2$ . At the same time it is observed that the power  $P_R$  increases when  $u_1$  increases from zero.

If  $u_1$  increases towards  $c_{u1}$ ,  $c_{u2}$  also increases and approaches to  $c_{u1}$  so that  $(c_{u1} - c_{u2})$  advances towards zero. In this case the power again approaches towards zero. This case corresponds to the run away speed.

*A closer examination shows that the transferred power  $P_R$  to the runner has its maximum value when  $c_{u2}$  is close to zero and accordingly  $u_1$  approximately like  $c_1/2$ .*

Regulation of the power means to regulate the discharge  $Q$  by adjustment of the needle position to larger or smaller openings of the nozzles. For constant angular speed the regulations really cause minor or no changes of the velocity diagrams.

### 2.2.3 Reaction turbines

Francis, Kaplan and Bulb turbines are the reaction turbines normally applied. The transfer of hydraulic energy into mechanical energy is principally similar in these turbines. However, the hydraulic design of Francis turbines differ so much from that of Kaplan and Bulb turbines that an interpretation of the energy transfer will be given for each of these two groups.

### Francis turbines

Fig. 2.3 shows an axial section through a Francis turbine with the guide vane cascade (G) and the runner (R). The runner is fastened to the turbine shaft (S).

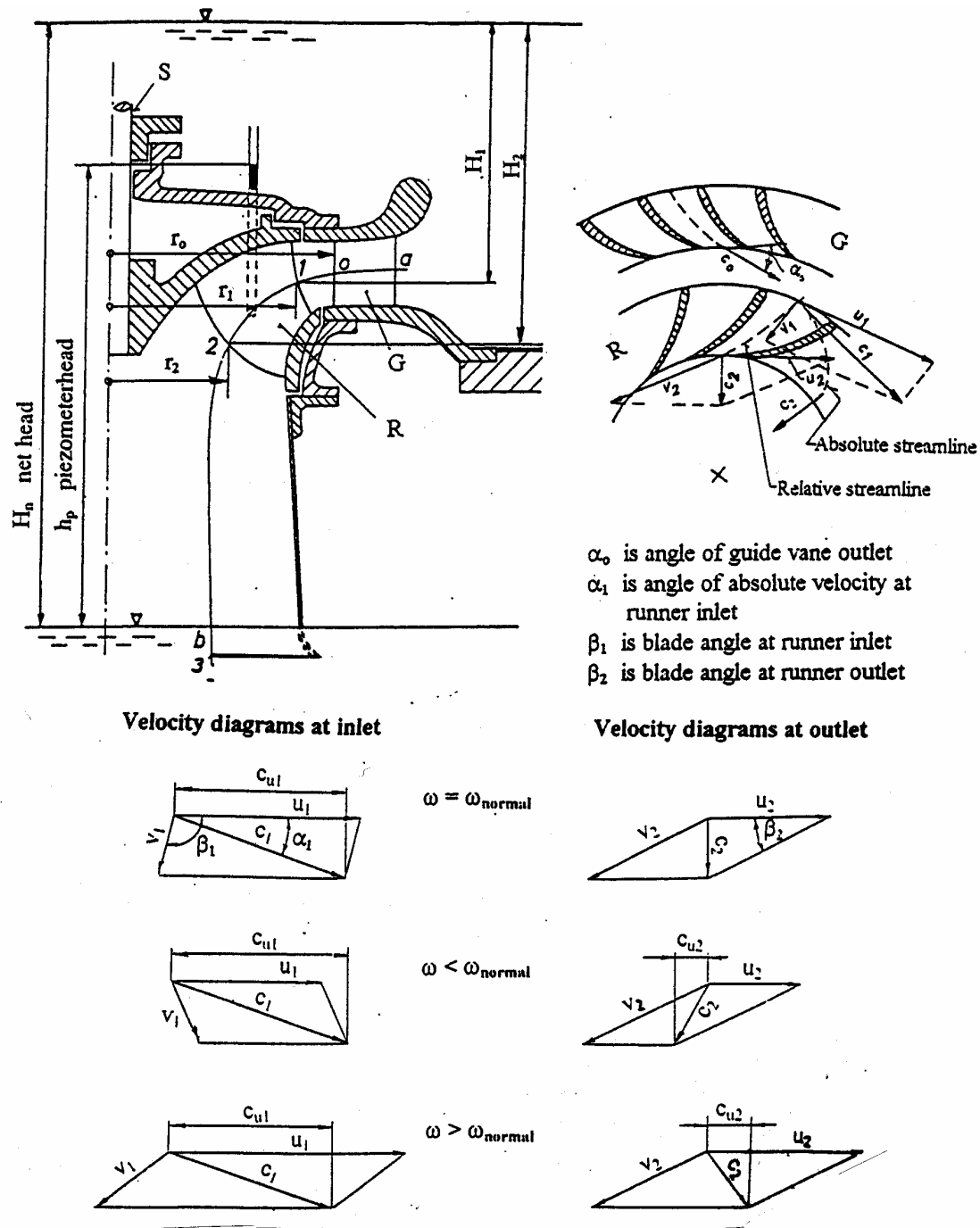


Fig. 2.3 Axial section through a Francis turbine. Velocity diagrams at inlet and outlet of the runner /3/

The energy transfer will be considered on the basis of the movement of one water particle along the coaxial stream surface of revolution having a contour (a - b), in the axial section through the turbine. To observe the movement of this particle through the turbine, a section across the guide

vanes and runner blades perpendicular to the paper plane in Fig. 2.3 is needed. This should be made along the contour (a - b), i.e., along (a - o) through the guide vane cascade and along (1 - 2) through the runner cascade.

However, the stream surface (a - b) in Fig.2.3 has a double curvature. Therefore the surface cannot be unfolded unless by a transformation from the real surface to a single curved surface. In practise this can be done by using conform transformation method because the angles of geometry then are kept unchanged from the real surface to the transformed surface.

The section through the runner cascade shown to the right on Fig. 2.3, is assumed to represent a conform transformation onto a conical surface tangential to the contour (1 - 2).

It is assumed that the turbine is installed at the bottom of an open reservoir filled with water up to a certain level above the guide vane cascade. The guide vane direction angle  $\alpha_o$  is assumed constant and the runner is rotating with a certain angular speed  $\omega$  and the water is filling all the runner canals completely.

The consideration is assumed to start with a water particle at the inlet edge of the guide vane cascade. Through the guide vane canal the water particle is assumed to follow the streamline in the middle of the canal width as shown on the figure. The guide vanes are designed so that the movement of the particle is changed from the radial direction at inlet to leave the outlet edge of the canal with a rather large velocity component in the peripheral direction. The outlet edge of the guide vanes is indicated with the index (o), and the *absolute* velocity of the water particle at this edge is accordingly denoted  $c_o$ . The direction of  $c_o$  is supposed to coincide with the direction of the vanes at the outlet of the guide vane cascade.

It is assumed that the water particle passes without friction through the ring chamber between the guide vane outlet and the inlet of the runner. Therefore it will keep an unchanged vortex momentum. That means  $rc_u$  is constant, and the relation between the rotational components  $c_{uo}$  and  $c_{u1}$  of the absolute velocities  $c_o$  and  $c_1$  respectively become

$$c_{u1} = c_{uo} \frac{r_o}{r_1} \quad (2.13)$$

where  $r_o$  is the radius to the guide vane outlet marked (o)  
 $r_1$  is the radius to the inlet of the runner marked (1)

The peripheral velocity of the runner corresponding to radius  $r_1$  is found by  $u_1 = r_1\omega$ .

Now the absolute velocity  $c_1$  and the peripheral velocity  $u_1$  are determined. The relative velocity  $v_1$  is then found as one side in the parallelogram where the peripheral velocity vector  $u_1$  is the other side and the absolute velocity vector  $c_1$  is the diagonal. These three velocity vectors are drawn in Fig. 2.3 and form the velocity diagram of the water particle at the inlet of the runner canal.

During the movement through the runner canal the particle changes its direction again as shown on the figure. By this deflection an impulse force is transferred by giving the runner a torque in the rotational direction.

The relative streamline through the runner canal is also drawn in the Fig.2.3. At the outlet edge of the canal on this streamline the relative velocity is denoted  $v_2$ . The figure shows that the relative velocity  $v_2$  has got a rather large peripheral component in the opposite direction of the rotation. The magnitude of  $v_2$  can be found by means of the continuity equation

$$v_2 a_2 = v_1 a_1 \quad (2.14)$$



where  $a_1$  and  $a_2$  are the areas of the runner blade canal taken perpendicular to the streamline at inlet and outlet of the canal respectively. The direction of  $v_2$  is the same as the outlet direction of the runner blades. The peripheral velocity  $u_2 = r_2\omega$  where  $r_2$  is the radius to the marked point (2) at the runner outlet.

The velocity diagram for the water particle at the outlet edge of the runner canal can be determined by drawing the parallelogram with the sides  $u_2$  and  $v_2$  from point (2) and thereafter the diagonal which is the resultant  $c_2$  drawn from the same point.

The passage of the water particle from point (1) to point (2) in the runner canal, needs a certain time interval for this movement, and simultaneously the runner rotates a certain angle. By drawing corresponding positions of the runner canal in the rotational direction and the position of the particle in the canal for some intermediate time intervals, the absolute path of the water particle is found. A such path is drawn on Fig. 2.3, and the absolute velocity vector is everywhere tangent to the absolute flow path.

The absolute as well as the relative movement of all particles in the water flow through the turbine will behave in the same way as described for the considered water particle. In a corresponding way the same impulse and torque is supposed to be transferred to the runner from all water particles.

The power transfer  $P_R$  to the runner from the water flow is then

$$P_R = \rho Q(u_1 c_{u1} - u_2 c_{u2}) \quad (2.15)$$

As mentioned for the impulse turbine,  $c_2^2/2$  represents the energy at outlet. However, during the passage of the draft tube a fraction of this energy is recovered by retardation of the flow velocity, but a flow friction loss occurs also in the draft tube which again means a slight reduction of the the recovered energy.

A discussion of Equation (2.15) may be carried out in the same way as done for the Pelton turbine. Examples are drawn in Fig. 2.3 of the velocity diagrams at inlet and outlet of the runner respectively for three angular velocities,  $\omega = \omega_{\text{normal}}$ ,  $\omega < \omega_{\text{normal}}$  and  $\omega > \omega_{\text{normal}}$ . A difference should however, be remarked that when the power is near maximum,  $u_1$  and  $c_{u1}$  nearly have the same magnitude, while  $u_1$  is approximately half of  $c_{u1}$  for the Pelton turbine.

For regulating the discharge  $Q$  of the turbine the width of the guide vane canals must be varied. An increase of  $Q$  means to adjust the guide vanes to a larger angle  $\alpha_0$  and a decrease of  $Q$  means an adjustment in the opposite direction. This regulation causes corresponding changes in the direction of the absolute velocity  $c_1$ . Accordingly the velocity diagrams will change.

Both the variation of the angular velocity  $\omega$  and the regulation of the discharge  $Q$  involve changes in the direction and magnitude of the relative velocity  $v_1$ . The relative velocity  $v_2$  varies accordingly in magnitude with the regulation of  $Q$ . Moreover the difference  $(u_1 c_{u1} - u_2 c_{u2})$  and thereby also the power transfer, is entirely dependent of these changes.

The most efficient power transfer however, is obtained for the operating condition when the relative velocity  $v_1$  coincide with blade angle  $\beta_1$  at the runner inlet and simultaneously the rotational component  $c_{u2} \approx 0$ . Therefore the hydraulic lay out of all reaction turbine runners are based on the data of rotational speed  $n$ , discharge  $Q$  and net head  $H_n$  for which the optimal efficiency is wanted.

### ***Kaplan and Bulb turbines***

The hydraulic design of Kaplan and Bulb turbine runners is quite similar while the flow direction through the guide vane cascades is radial in Kaplan turbines and approximately axial in Bulb turbines. This means no significant difference for an interpretation of the flow through these turbines. Therefore an illustration of the flow through a Kaplan is valid also for a Bulb turbine.

Fig. 2.4 shows an axial section through a Kaplan turbine with the guide vane cascade (G), runner (R) and the shaft (S).

In the same way as for Francis turbines the consideration of the flow through the Kaplan turbine is based on the movement of one water particle along the flow surface with the contour (a - b) as shown on Fig. 2.4. This particle is given a flow direction in the guide vane canal before it enters the runner canal. The particle movement through the guide vane canal and runner canal is shown to the right in the figure. The canal section across guide vane cascade is radial (perpendicular to the plane) along contour (a - o) and across the runner blades the section is cylindrical along the contour (1 - 2).

The considerations are further based on a constant guide vane direction angle  $\alpha_o$  and constant angular velocity  $\omega$ . The particle flow through the turbine is quite analogous to that described for the Francis turbine. Therefore the description is focused mainly on the velocity diagrams.

The fluid flow in the axisymmetrical hollow space between outlet of the guide vane canal marked (o), and the inlet of the runner marked (1), is denoted as free vortex. The flow is again assumed free of losses along the flow path. The relation between the rotational component  $c_{uo}$  of the absolute velocity  $c_o$  and the rotational component  $c_{u1}$  of the absolute velocity  $c_1$  then becomes

$$c_{u1} = c_{uo} \frac{r_o}{r_1} \quad \text{or} \quad c_{u1} = c_{uo} \frac{u_o}{u_1} \quad (2.16)$$

The peripheral velocities are  $u_1 = r_1\omega$  and  $u_2 = r_2\omega$ . By the given angle  $\alpha_o$  at the outlet of the guide vane canal and the angle  $\beta_2$  at the outlet of the runner canal all data are now prepared for drawing of the velocity diagrams at inlet and outlet of the runner. The drawing of these diagrams is as described for Francis turbines, and in Fig. 2.4 examples are shown for three different angular speeds,  $\omega = \omega_{\text{normal}}$ ,  $\omega < \omega_{\text{normal}}$  and  $\omega > \omega_{\text{normal}}$ .

The designation  $\omega = \omega_{\text{normal}}$  means again the rotational speed for which the turbine obtain the lowest energy loss at outlet represented mainly by  $c_2^2/2$ . This is also the operating condition for which the turbine obtain the highest hydraulic efficiency for the given angle  $\alpha_o$  of the guide vane canal.

The movement that is considered for a single water particle through the turbine, is an illustration also for all the other particles in the total water flow. As mentioned for Francis turbines that is an assumption for Kaplan and Bulb turbines as well that all particles transfer the same impulse and torque to the runner. The power transfer from the flow is therefore generally expressed by Equation (2.15)

$$P_R = \rho Q(u_1 c_{u1} - u_2 c_{u2})$$

A look at Fig. 2.4 indicate also that the peripheral velocity  $u_2 = u_1$ . The power equation then becomes

$$P_R = \rho Q u_1 (c_{u1} - c_{u2}) \quad (2.17)$$

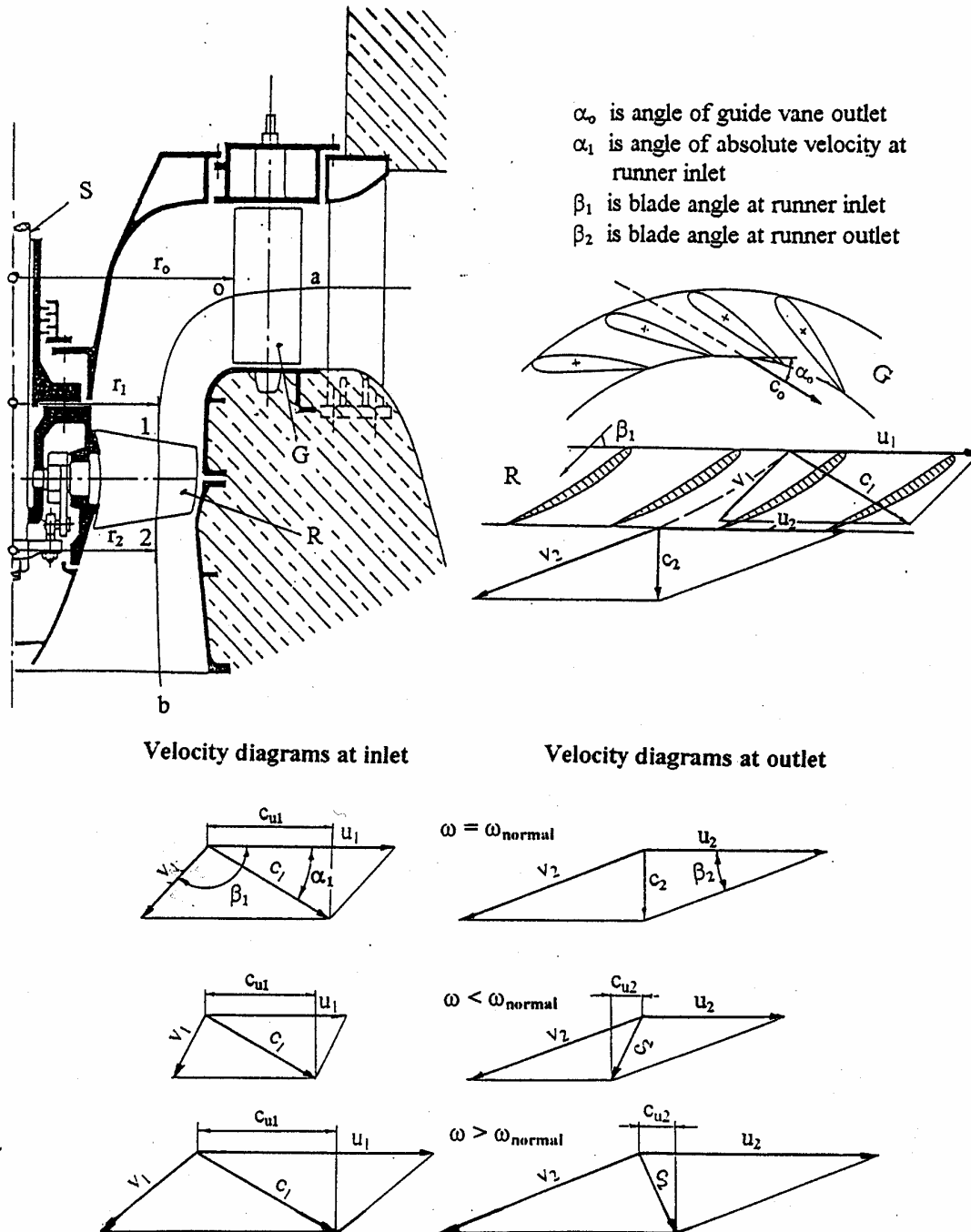


Fig. 2.4 Axial section through a Kaplan turbine. Velocity diagrams at inlet and outlet /3/

An interpretation of power regulation by changing the discharge  $Q$  with adjustment of the openings of the guide vane cascade follows the same reasoning as for Francis turbines. However,

in Kaplan and Bulb turbines the direction of the runner blades can also be varied. This property will be further interpreted in Chapter 3.

## 2.2.4 The main equation of turbines

### *Losses along a flow path*

The flow through turbines is exposed to energy losses. In reaction turbines the flow friction and change of flow directions in the guide vane cascade, the runner and the draft tube mainly cause these losses. According to the turbulent flow conditions these energy losses are defined <sup>/4/</sup> by

$\zeta_1 \frac{c_1^2}{2}$  is the energy losses in the guide vane cascade

$\zeta_2 \frac{v_2^2}{2}$  is the energy losses in the runner

$(1 + \zeta_3) \frac{c_3^2}{2}$  is the energy losses in the draft tube

The coefficients  $\zeta_1$ ,  $\zeta_2$  and  $\zeta_3$  are assumed as constants <sup>/4/</sup>, but their values depend on the operating conditions of the turbine. In general this dependence behaves so that  $\zeta_1$  and  $\zeta_2$  have a smaller variation than  $\zeta_3$  when operating conditions changes. Values of general validity of these coefficients cannot be given. However, at favourable operating conditions for a reaction turbine one can estimate values of  $\zeta_1$  and  $\zeta_2$  in the region 0.06 - 0.15 and of  $\zeta_3 = 0.1 - 0.3$ .

In addition to the losses by friction and changed directions, so-called impact losses can occur at the inlet of the runner. These losses which are designated by  $E_I^2$ , are introduced when the relative velocity vector of the flow enters the runner with another direction than the inlet direction of the blade.

The total sum of the losses along the flow path is

$$h_L = \frac{1}{2} [\zeta_1 c_1^2 + \zeta_2 v_2^2 + (1 + \zeta_3) c_3^2 + E_I^2] \quad (2.18)$$

The available net head for the turbine is designated  $H_n$ . The specific energy head transferred to the runner is then

$$h_R = H_n - h_L \quad (2.19)$$

### *Note*

When the operating conditions by regulations deviate quite a lot from the design operating conditions, the same assumptions of the magnitude of the loss coefficients along the flow paths do not hold completely.

### *The main equation of turbines*

The total available power of a plant is

$$P_n = \rho Q g H_n \quad (2.20)$$

The net head  $H_n$  is defined at the inlet of the turbine referred to the level of the tail water of

reaction turbines or the outlet of the nozzle of a jet turbine.

The power transfer from the fluid to the turbine runner is

$$P_R = \rho Q(u_1 c_{u1} - u_2 c_{u2}) \quad (2.21)$$

The ratio between these powers

$$\eta_h = \frac{P_R}{P_n} = \frac{1}{gH_n}(u_1 c_{u1} - u_2 c_{u2}) \quad (2.22)$$

which is defined as *hydraulic efficiency*.

The following rearrangement of Equation (2.22) is called the *main turbine equation*

$$\eta_h H_n = \frac{1}{g}(u_1 c_{u1} - u_2 c_{u2}) \quad (2.23)$$

Another version of the main turbine equation is obtained by substituting for  $u_1 c_{u1}$  and  $u_2 c_{u2}$  from the velocity triangles respectively

$$H_n = (1 + \zeta_1) \frac{c_1^2}{2g} - \frac{c_2^2}{2g} + (1 + \zeta_2) \frac{v_2^2}{2g} - \frac{v_1^2}{2g} + \frac{u_1^2}{2g} - \frac{u_2^2}{2g} + (1 + \zeta_3) \frac{c_3^2}{2g} + \frac{E_1^2}{2g} \quad (2.24)$$

This last version expresses the total sum of the energy transfer in the runner and the head losses in the guide vane cascade, the runner and the draft tube as equal to the net head  $H_n$ . For Pelton turbines however, it should be remarked that the two last terms in equation (2.24) are omitted, i.e., these terms do not occur in the energy conversion analysis for these turbines..

## 2.3 A brief outline of the hydraulic design of turbines

### *The axial flow out of reaction turbines*

By the considerations of the flow along the stream surface of revolution having a contour given in Fig. 2.3, the main obtained result was the *main turbine equation* which is expressed in several equations (2.23 - 2.24). For the conditions being the design basis of a turbine, the major objective of the design is to obtain the same hydraulic efficiency for any flow surface of similar properties as the surface with the contour a - b, through the turbine over the whole width of the flow space.

To deal with this design process in detail is not the aim in this text. Therefore only a brief description of it will be given. Principally this will be about the hydraulic designing.

One of the first steps in the design is to form the contours of the axial section of the turbine. It may start with a trial of the form and thereafter a computational verification of the distribution of the meridional flow velocity through the axial section. The determination of the meridional velocity profile is based on the law of irrotational flow

$$\frac{dc_z}{dn} = -\frac{c_z}{R} \quad (2.25)$$

where  $c_z$  is the meridional component of the absolute velocity

$n$  is the length parameter of the width perpendicular to the flow surface

$R$  is the radius of the curvature of the flow path contour

When the velocity profile is determined according to equation (2.25) a control of how far this is correct, has to be done by use of the continuity

$$dQ = 2\pi r c_z dn \quad (2.26)$$

equation where  $r$  is the radius to the actual position on the flow surface. The examination of a correct flow distribution may be done in the following way. The axial flow section is thought to

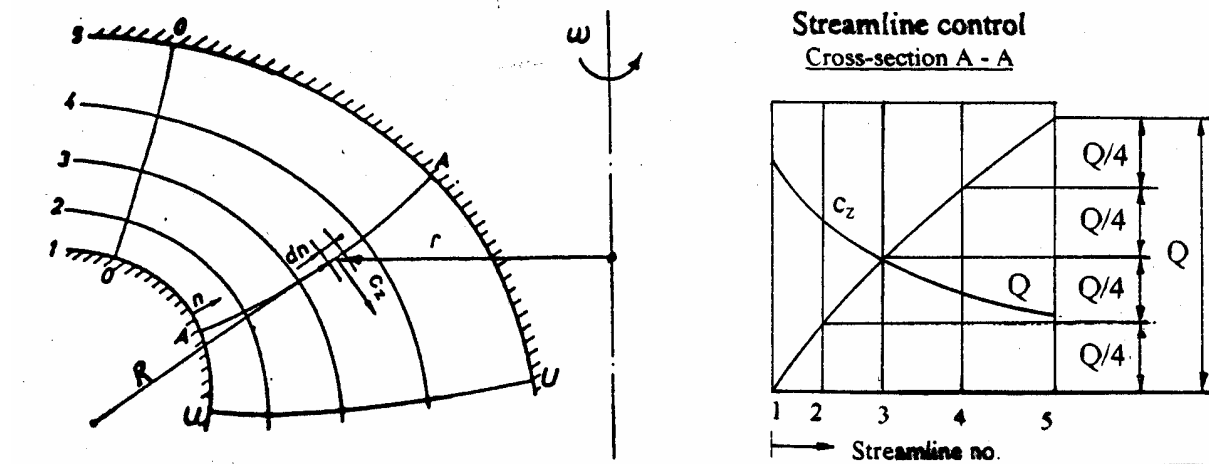


Fig. 2.5 Axial section of a Francis runner divided in four subsections by five numbered paths /3/

be divided in a certain number of coaxial subsections through the turbine as shown on Fig. 2.5, where the number of subsections is four. Each of these subsections shall represent a fraction of the total discharge  $Q$ , that means one quart of  $Q$ . To obtain this the velocity profiles of  $c_z$  from equation (2.25) are tested and corrected by use of equation (2.26) until the discharge is the same in all the subsections. Further it is important to check that the shape of the axial section performs an even or smooth meridional velocity  $c_z$  so that it does not increase or decrease disorderly in magnitude along the flow path.

### Runner canals and blade forms

The shaping of the blades and the runner canals is a topic task for skilled designers. This process is again a combination of theoretical computations, experiences, certain criteria and rules.

As one of the first steps an estimation of the number of blades has to be done. The turbine manufacturers have normally specific criteria of their own for this choice. But there are also some general requirements that have to be satisfied. For example the smallest openings between blades must be large enough to let expected firm contamination in the flow run freely through. Further the designers have to give the blades a shape based on control procedures so that the energy conversion in the runner for the whole flow path is satisfied.

To have an idea of some details in this type of design work, it may be beneficial to have a look at a section through a few pairs of blades and the canals they constitute. At first a glance back on Figs. 2.3 or 2.4, gives examples of blade forms and canals. In some of these canals a streamline is drawn. This streamline served as basis for the considerations being carried out on the kinematics of the hydraulic energy transfer in reaction turbines. Furthermore this streamline was assumed to represent the average velocity in the corresponding cross-sections along the canals. Now a look in more detail on the flow conditions through the runner canals will show velocity

distributions over the cross-sections as indicated on Fig. 2.6. To determine the velocity profile in the respective cross-sections of the canals, the following differential equation may be used

$$\frac{dv}{dn} = -\frac{v}{R} - 2\omega \cos \delta \quad (2.27)$$

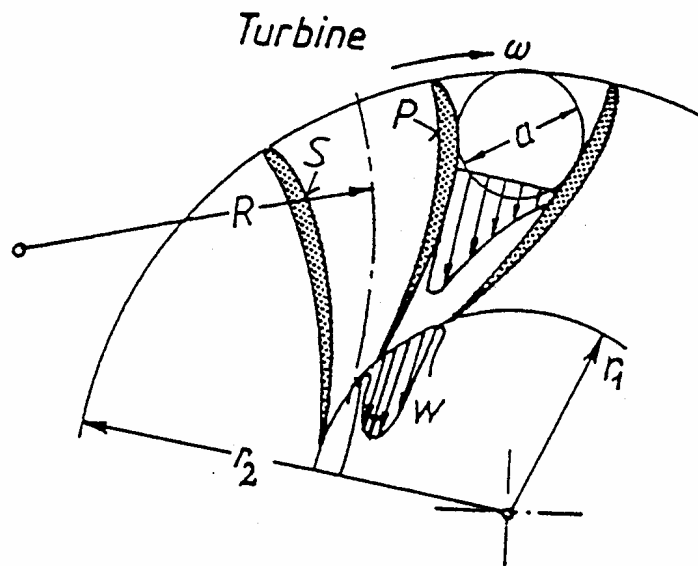


Fig. 2.6 Section through runner blades. An example of the skew velocity profile

where

$v$  is the relative velocity in the runner canal

$R$  is the curvature radius of the relative streamline

$\omega$  is the angular speed of the turbine

$\delta$  is the angle between the turbine axis and the perpendicular to the cone surface in which the section through the runner blade canal are depicted

$n$  is the length parameter of the width of the canal perpendicular to the streamlines.

The estimations of the velocity profiles according to equation (2.27) have again to be examined by means of the equation of continuity.

In general one can say that the blades should never be formed with particular sharp curvature. It is considered as ideal to give the blades a smooth form between the inlet and the outlet of the canals. This may often hit upon opposing requirements that lead to judgements and compromises between blades of short length and sharp curvature on one side and long blades with a smooth curvature on the other. Long canals are exposed to larger viscous friction losses than the short ones, while short canals are exposed to larger impulse and impact losses. The best solution is to be found somewhere in between these extreme limits. In such cases skilled designers are of course the right experts to do the job.

### **Pelton buckets**

The design of buckets of Pelton runners<sup>/2/</sup> involves preliminary drafts of the hydraulic form of the buckets followed by computational and experimental examination and adjustment of the draft<sup>/2/</sup>.

To have an idea of how the hydraulic modelling of the buckets may be done, Fig. 2.7 shows a calculated flow picture over a Pelton bucket at a certain moment. This analysis is based on the fact that the accelerations of the fluid particles must be perpendicular to the surface of the water flow in the bucket at any moment. The traces of fluid particles can then be found as shown on the figure. The following equations are used to find the accelerations and traces:

$$a_x = \frac{d^2x}{dt^2} + \omega^2 R \cos \phi + 2\omega u_y \quad (2.28)$$

where  $\cos \phi = x/R$  and  $R$  = runner radius

$$a_y = \frac{d^2y}{dt^2} + \omega^2 R \sin \phi + 2\omega u_x \quad (2.29)$$

$$a_z = \frac{d^2z}{dt^2} \quad (2.30)$$

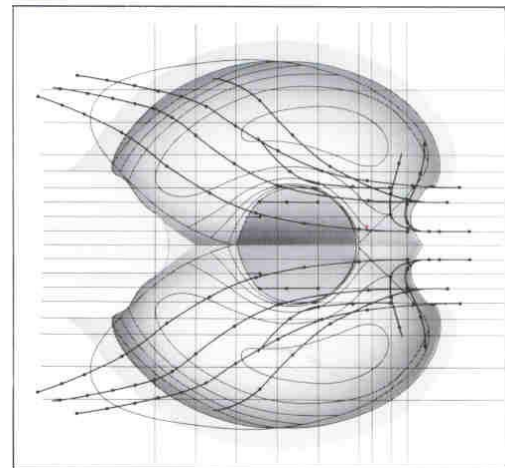
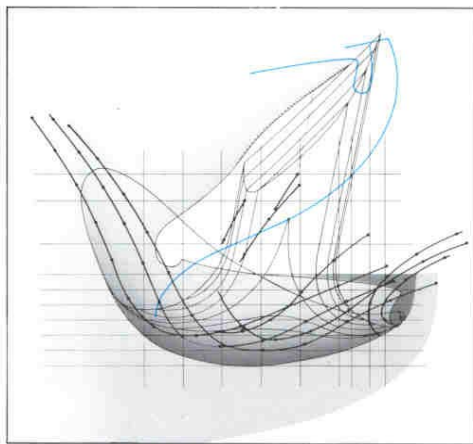


Fig. 2.7 Flow traces over a Pelton bucket at a certain moment (calculated) /2/

## 2.4 Efficiency

The equation of hydraulic efficiency, Eq. (2.22), expresses

$$\eta_h = \frac{P_R}{P_n}$$

The power transfer to the runner is further exposed to additional losses before the resulting power  $P$  is transferred to the generator shaft. These losses are composed of mechanical friction in the bearings and stuffing boxes, viscous friction from the fluid between the outside of the runner and the covers of the reaction turbines and ventilation or air friction losses of the runner in impulse turbines.

Through the space between the covers and the outside of the runner a leakage flow also passes according to the clearances of the labyrinth seals, from the inlet rim to the suction side of the runner. Some energy is also required for operation of the turbine governor, tapping water for sealing boxes, ejectors and cooling of bearings and the governor oil.



On account of all these losses the turbine efficiency is always lower than the hydraulic efficiency. Therefore, at the discharge  $Q$  and the power  $P$  transferred from the turbine shaft to generator shaft, the turbine efficiency is

$$\eta = \frac{P}{P_n} = \frac{P}{\rho Q g H_n} \quad (2.31)$$

Usually the maximum efficiency point which is represented by the best operating conditions, is reaching values of say  $\eta = 0.93$  to  $0.95$  of the larger and best reaction turbines. Corresponding values estimated for the hydraulic efficiency  $\eta_h = 0.95$  to  $0.97$ . For the best Pelton turbines  $\eta_{\max}$  reaches values about  $0.92$

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