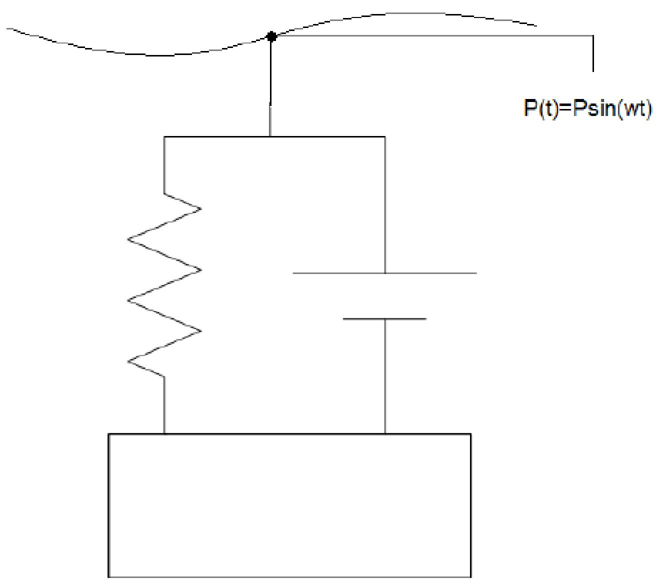
B.2 Vibration Calculations

Vibration Analysis

The motion is in the vertical direction only
The motion at point P is the input to the system
The vertical motion x(t) of the body is the output



Equations of Motion

$$m\ddot{x} + b(\dot{x} - \dot{p}) + k(x - p) = 0$$

$$m\ddot{x} + b\dot{x} - b\dot{p} + k\dot{x} - kp = 0$$

$$m\ddot{x} + b\dot{x} + kx = b\dot{p} + kp$$

$$(ms^2 + bs + k)\,X(s) = (bs + k)P(s)$$

$$\frac{X(s)}{P(s)} = \frac{b \cdot s + k}{m \cdot s^2 + b \cdot s + k}$$

Sinusoidal Transfer Function

$$\frac{X(j\omega)}{P(j\omega)} = \frac{bj\omega + k}{-m\omega^2 + bj\omega + k}$$

$$TR = \frac{amplitude\_of\_the\_system}{amplitude\_of\_the\_output}$$

$$TR = \frac{|X(j\omega)|}{|P(j\omega)|} = \frac{\sqrt{b^2 \cdot \omega^2 + k^2}}{\sqrt{\left(k - m \cdot \omega^2\right)^2 + b^2 \omega^2}}$$

$$\frac{k}{m} = \omega_n^{\ 2}$$

$$\frac{b}{m} = 2 \cdot \zeta \cdot \omega_n$$

Transmisibility is in terms of the damping ratio and the undamped natural frquency ωn and $\beta = \omega / \omega n$

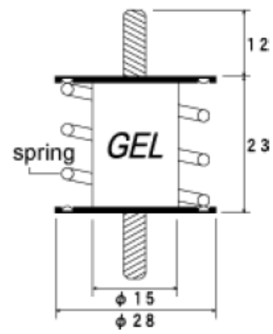
$$TR = \frac{\sqrt{1 + (2 \cdot \zeta \cdot \beta)^2}}{\sqrt{\left(1 - \beta^2\right)^2 + (2 \cdot \zeta \cdot \beta)^2}}$$

Calculating the values of each component:
We will assume that the system will be a total of 40lb

Converting from English to SI units $40\text{lb} = 18.144\text{kg}$

Spring Constant $k := 16600\frac{N}{m}$

$k = 1.66\times 10^{4}\frac{N}{m}$



$b := 4000\frac{N\cdot s}{m}$

$\omega := 69.115\frac{rad}{s}$

$m := 18.144\text{kg}$

harmonic displacement $x := .002$:

$45\cdot\frac{lb}{in} = 803.609\frac{kg}{m}$

natural frequency $\omega_n := \sqrt{\frac{k}{m}}$

damping ratio $\zeta := \frac{b}{2\cdot m\cdot \omega_n}$

$\zeta_1 := 0..5$

frequency ratio $\beta := \frac{\omega}{\omega_n}$

$\beta = 2.285$

$$TR := \sqrt{\frac{1 + (2 \cdot \zeta \cdot \beta)^2}{\left(1 - \beta^2\right)^2 + (2 \cdot \zeta \cdot \beta)^2}}$$

Solving for the Amplitude of absolute acceleration

$$X := x \cdot \sqrt{\frac{1 + (2 \cdot \zeta \cdot \beta)^2}{\left(1 - \beta^2\right)^2 + (2 \cdot \zeta \cdot \beta)^2}}$$

$$X = 2.428 \times 10^{-3}$$

Solving for the displacement of the projector relative to the support

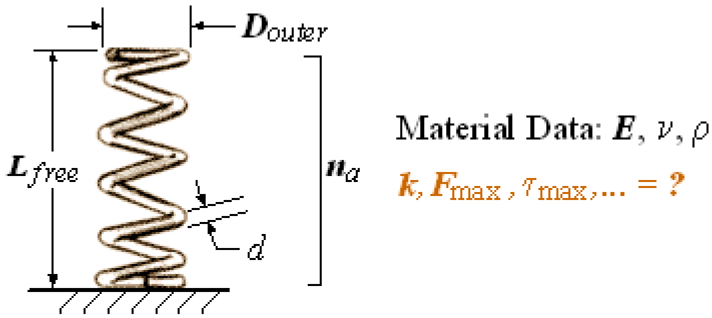
$$Z := x \cdot \frac{\beta^2}{\sqrt{\left(1 - \beta^2\right)^2 + (2 \cdot \beta \cdot \zeta)^2}}$$

Displacement $\quad Z = 7.597 \times 10^{-4}$

The realative displacement to the support in the Z direction is 0.76 mm
which is a large improvement.

This calculation was done at www.efunda.com

In determining the total number of coils in the spring, the calculator assumes that the ends of the spring are squared.



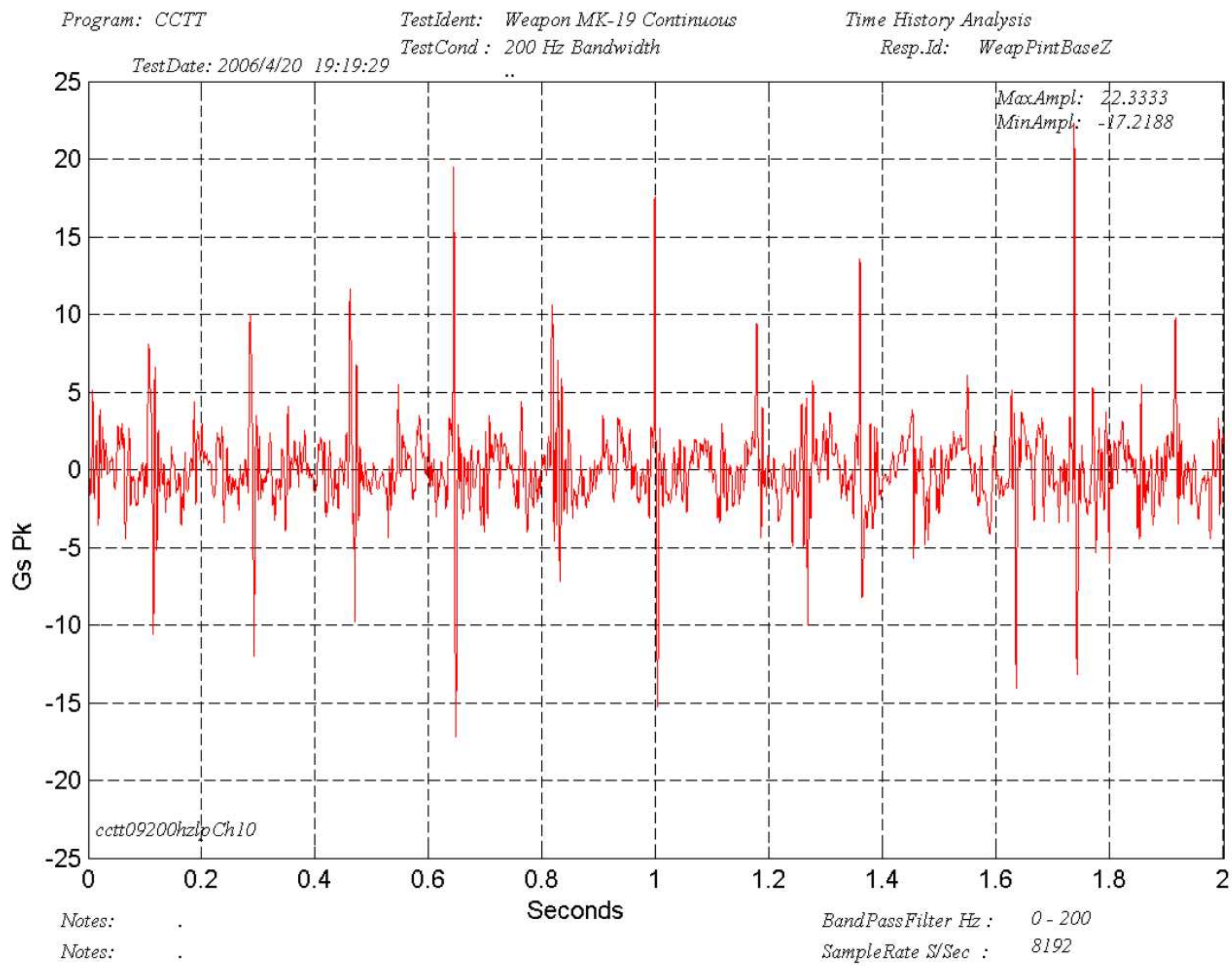
Inputs

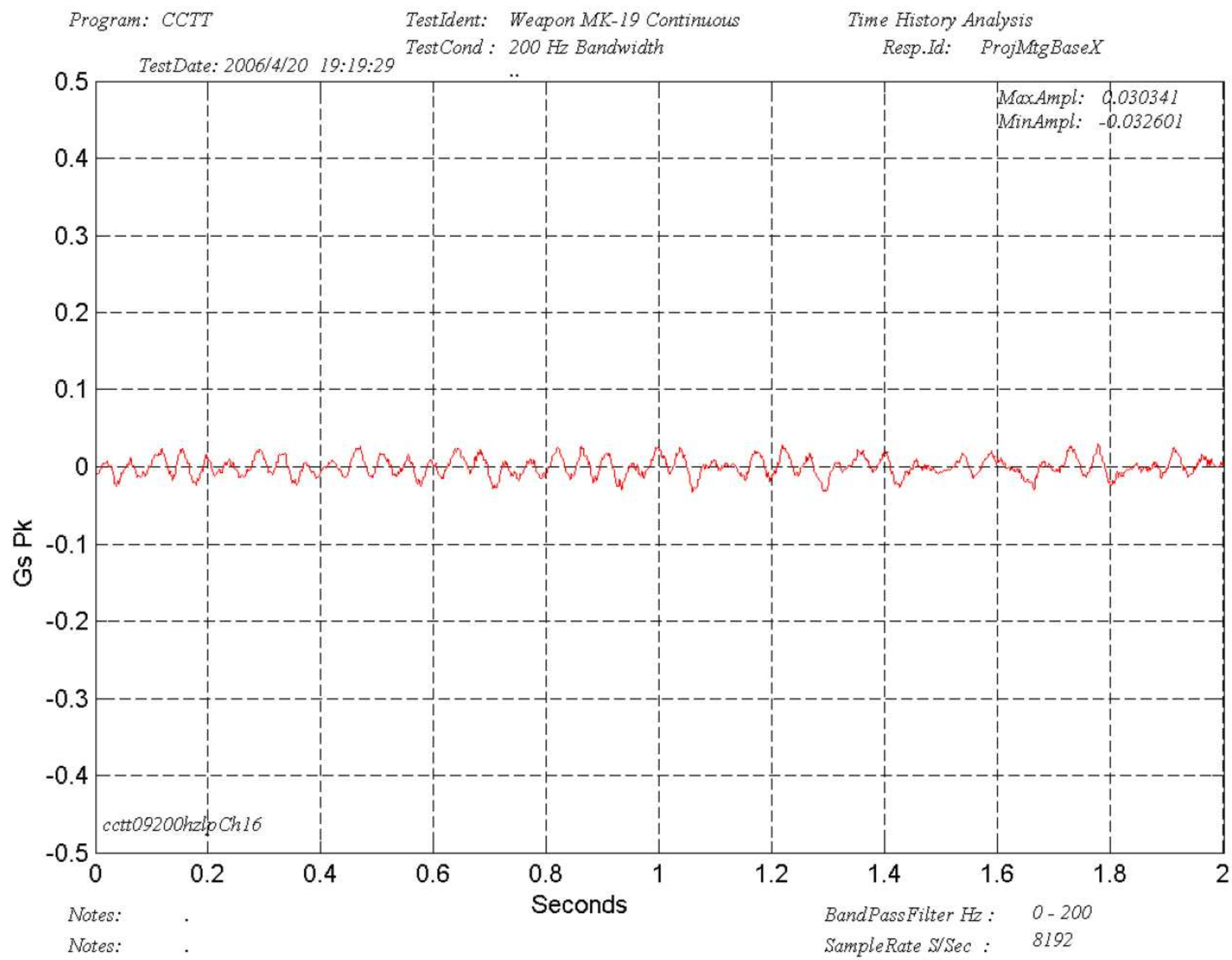
Diameter of spring wire, d:	3	mm
Outer diameter of spring, D_{outer}:	28	mm
Free length of spring, L_{free}:	23	mm
Number of active coils, n_a:	3	
Youngs modulus of material, E:	200	GPa
Poisson ratio of material, ν:	0.3	
Density of material, ρ:	7500	kg/m^3

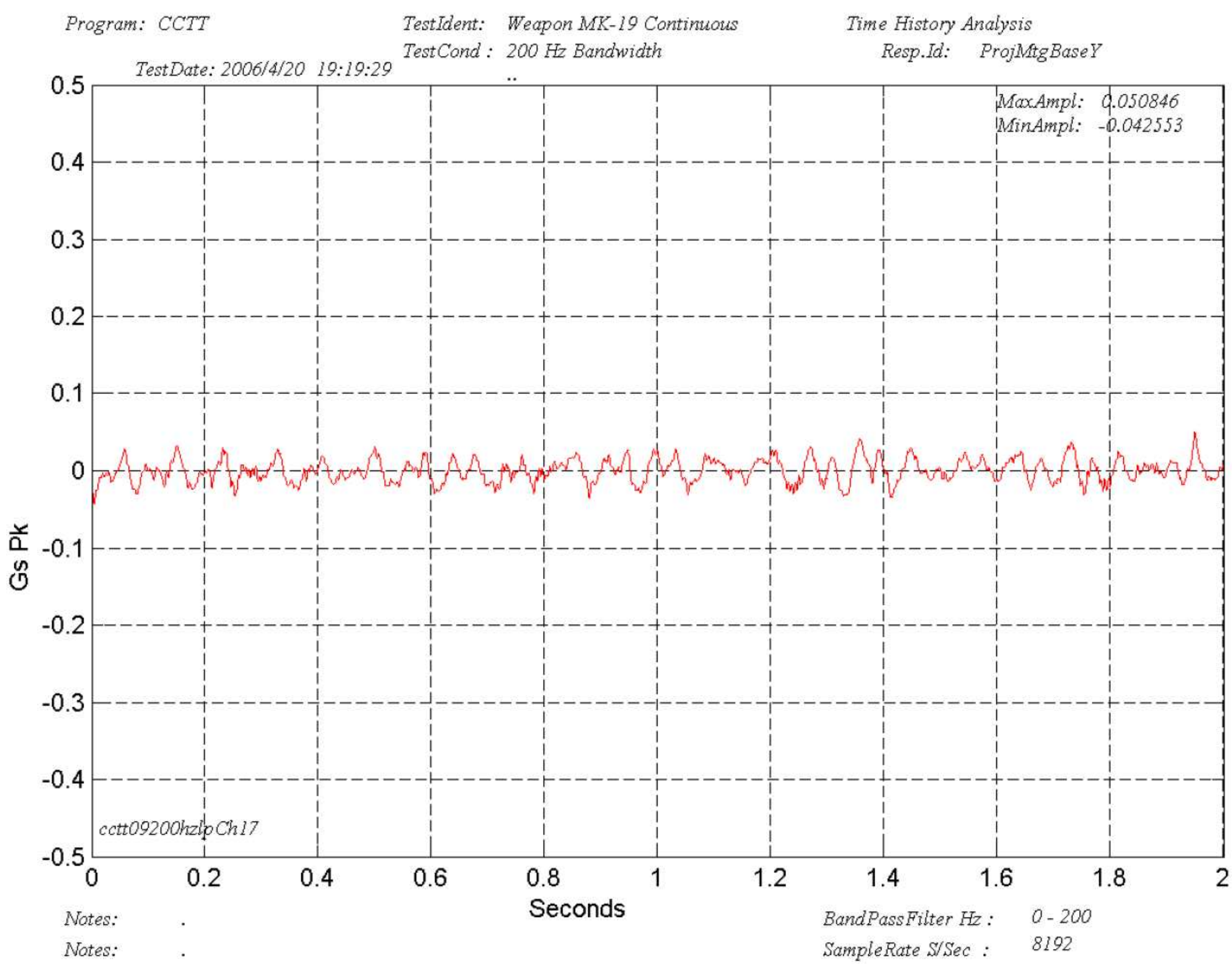
Answers

Spring constant, k:	1.66×10^4 N/m	N/m
Maximum load possible, F_{max}:	133 N	N
Maximum shear stress possible, τ_{max}:	3.69×10^5 kPa	kPa
Maximum displacement possible, L_{def}:	0.800 cm	cm
Length of wire required to make spring:	39.4 cm	
Solid height:	1.50 cm	
Distance between coils in free spring:	0.767 cm	
Rise angle of coils:	5.58 deg	
Lowest spring resonant frequency, f_{res}:	446 Hz	Hz
Shear modulus of material, G:	76.9 GPa	GPa
Mass of spring:	0.0209 kg	kg

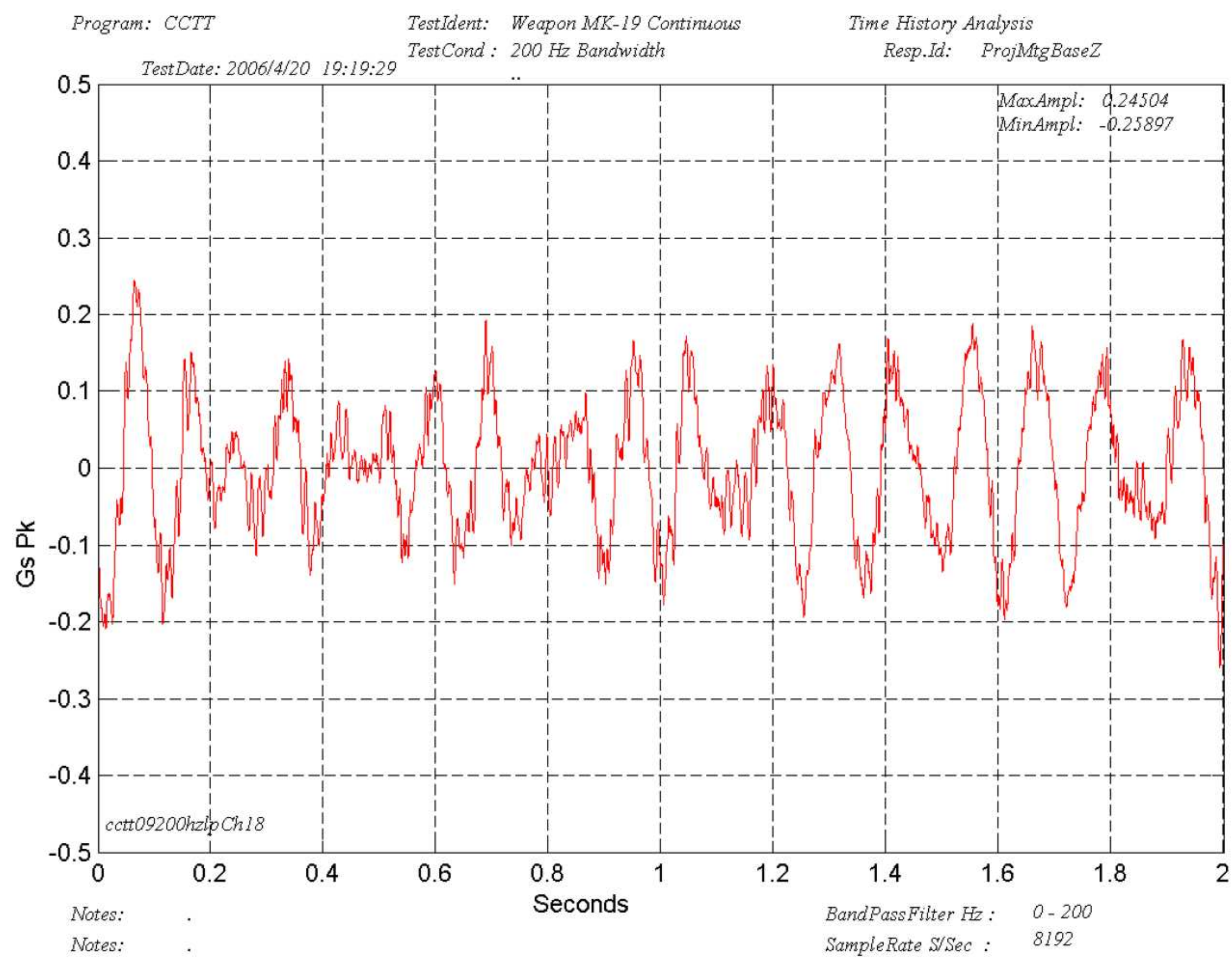
Select desired output units for next calculation.

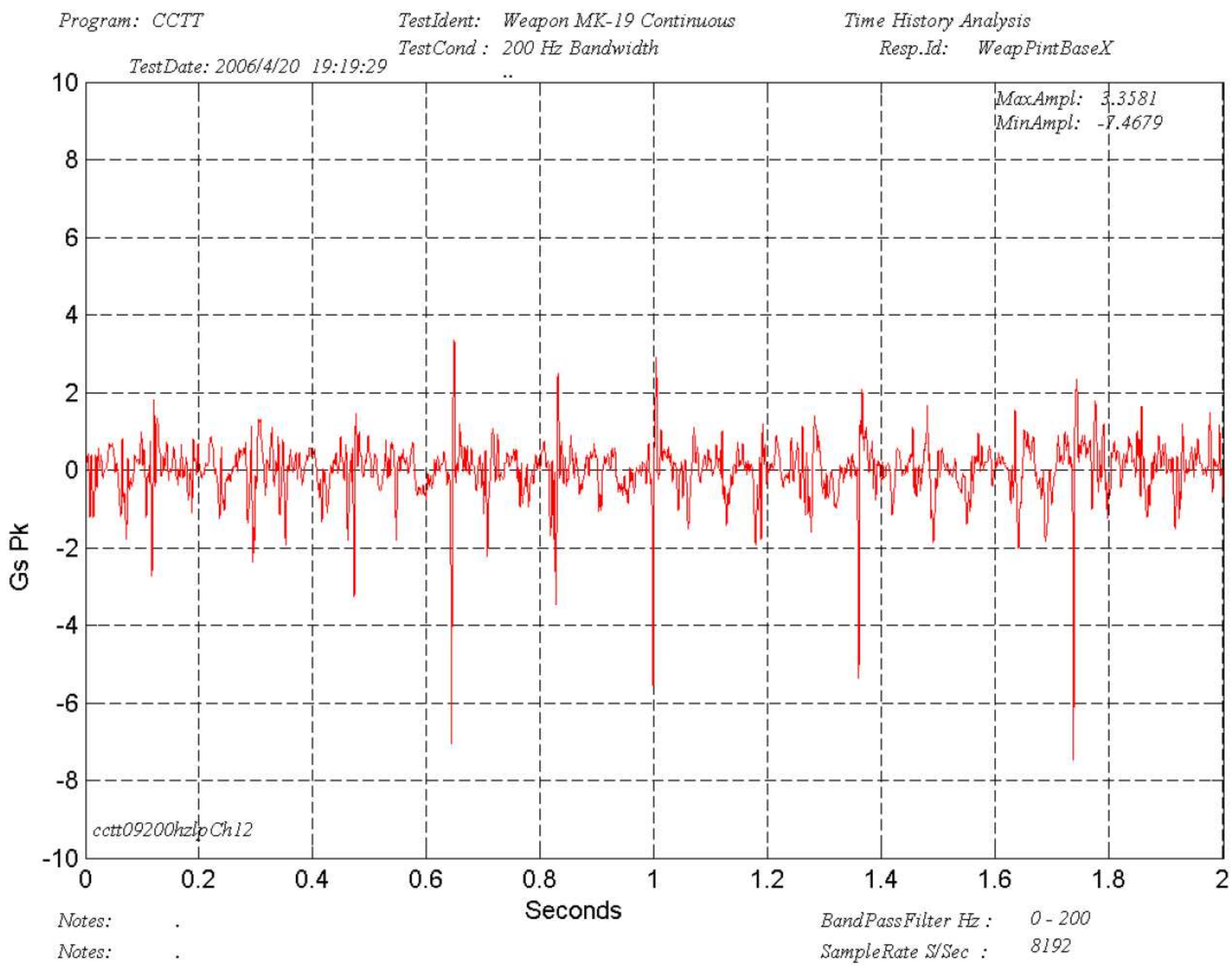

Program: CCTT
TestIdent: Weapon MK-19 Continuous
TestCond : 200 Hz Bandwidth
Time History Analysis
Resp.Id: WeapPintBaseZ
TestDate: 2006/4/20 19:19:29
MaxAmpl: 22.3333
MinAmpl: -17.2188
Gs Pk
25
20
15
10
5
0
-5
-10
-15
-20
-25
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Seconds
cctt09200hzlpCh10
Notes: .
Notes: .
BandPassFilter Hz : 0 - 200
SampleRate S/Sec : 8192

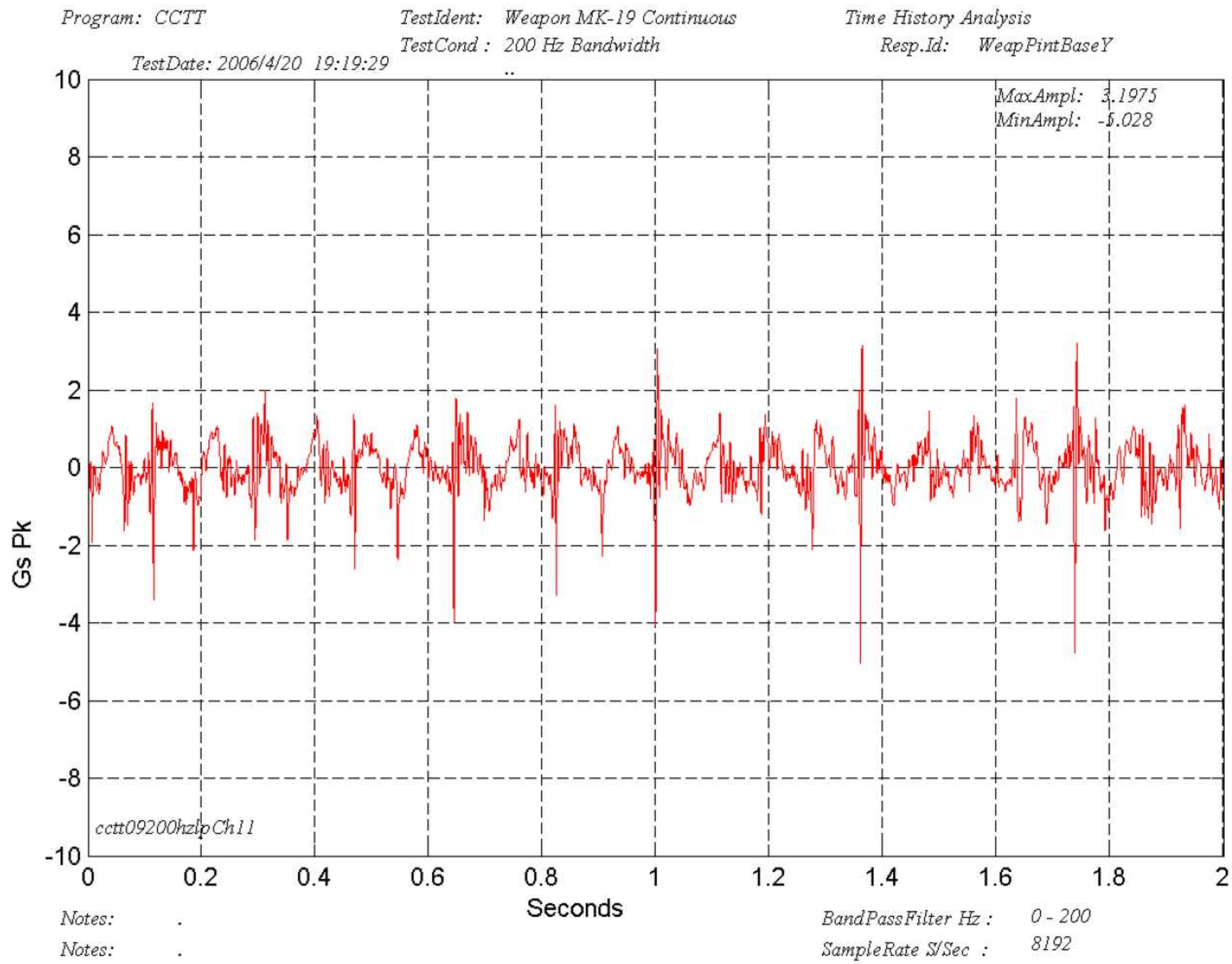

Program: CCTT
TestIdent: Weapon MK-19 Continuous
Time History Analysis
TestDate: 2006/4/20 19:19:29
TestCond : 200 Hz Bandwidth
Resp.Id: ProjMtgBaseX
MaxAmpl: 0.030341
MinAmpl: -0.032601
Gs Pk
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
cctt09200hzlpCh16
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Seconds
Notes: .
Notes: .
BandPassFilter Hz : 0 - 200
SampleRate S/Sec : 8192



Program: CCTT
TestIdent: Weapon MK-19 Continuous
Time History Analysis
TestCond : 200 Hz Bandwidth
Resp.Id: ProjMtgBaseY
TestDate: 2006/4/20 19:19:29
MaxAmpl: 0.050846
MinAmpl: -0.042553
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
Gs Pk
cctt09200hzlpCh17
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Seconds
Notes: .
Notes: .
BandPassFilter Hz : 0 - 200
SampleRate S/Sec : 8192


Program: CCTT
TestDate: 2006/4/20 19:19:29
TestIdent: Weapon MK-19 Continuous
TestCond : 200 Hz Bandwidth
Time History Analysis
Resp.Id: ProjMtgBaseZ
MaxAmpl: 0.24504
MinAmpl: -0.25897
Gs Pk
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
cctt09200hzlpCh18
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Seconds
Notes: .
Notes: .
BandPassFilter Hz : 0 - 200
SampleRate S/Sec : 8192


Program: CCTT
TestIdent: Weapon MK-19 Continuous
Time History Analysis
TestCond : 200 Hz Bandwidth
Resp.Id: WeapPintBaseX
TestDate: 2006/4/20 19:19:29
MaxAmpl: 3.3581
MinAmpl: -7.4679
10
8
6
4
2
0
-2
-4
-6
-8
-10
Gs Pk
cctt09200hzlpCh12
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Seconds
Notes: .
Notes: .
BandPassFilter Hz : 0 - 200
SampleRate S/Sec : 8192



Program: CCTT
TestIdent: Weapon MK-19 Continuous
Time History Analysis
TestCond : 200 Hz Bandwidth
Resp.Id: WeapPintBaseY
TestDate: 2006/4/20 19:19:29
MaxAmpl: 3.1975
MinAmpl: -5.028
Gs Pk
Seconds
cctt09200hzlpCh11
Notes: .
Notes: .
BandPassFilter Hz : 0 - 200
SampleRate S/Sec : 8192