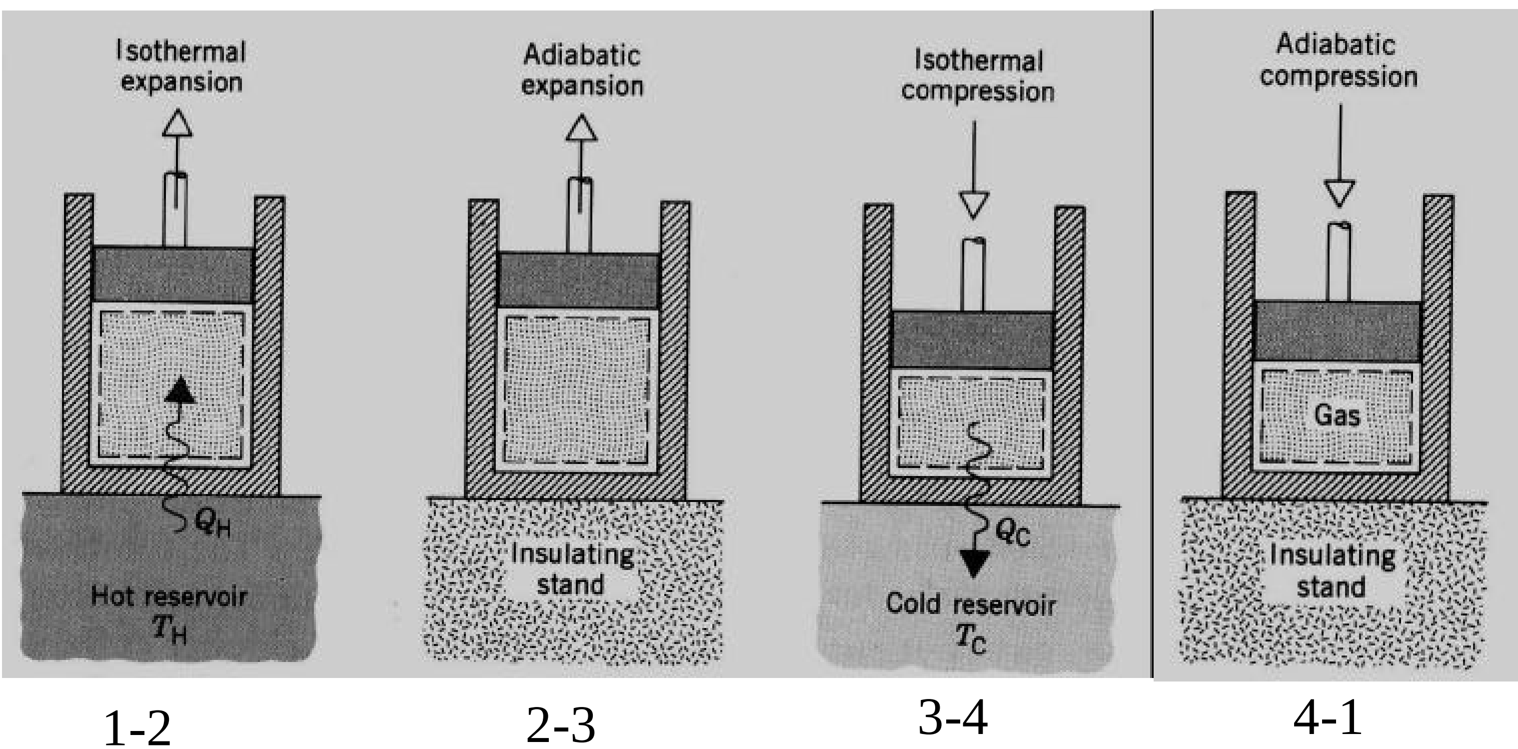
The Carnot Cycle

• Idealized thermodynamic cycle consisting of four reversible processes (any substance):

- Reversible isothermal expansion (1-2, T_H=constant)
- Reversible adiabatic expansion (2-3, Q=0, $T_H \rightarrow T_L$)
- Reversible isothermal compression (3-4, T_L=constant)
- Reversible adiabatic compression (4-1, Q=0, $T_L \rightarrow T_H$)



The Carnot Cycle-2

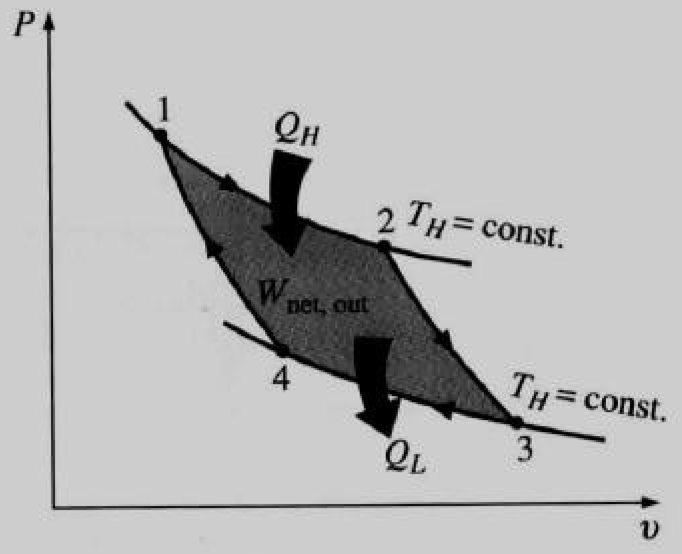
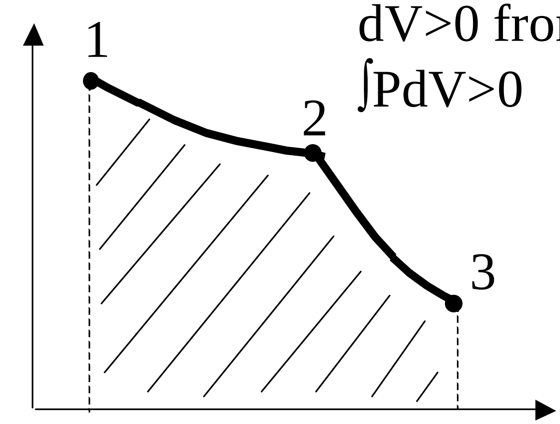


FIGURE 5-44
P-v diagram of the Carnot cycle.

Work done by gas = ∫PdV, area under the process curve 1-2-3.

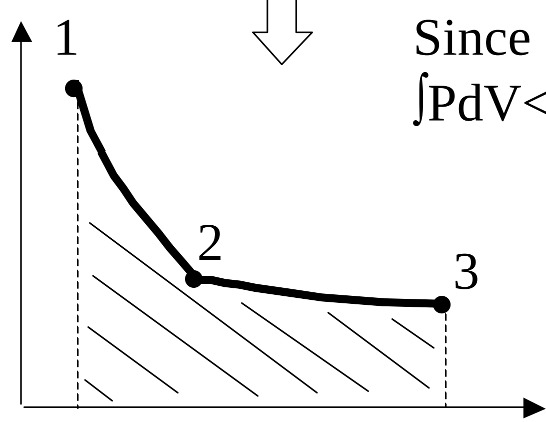
dV>0 from 1-2-3
∫PdV>0

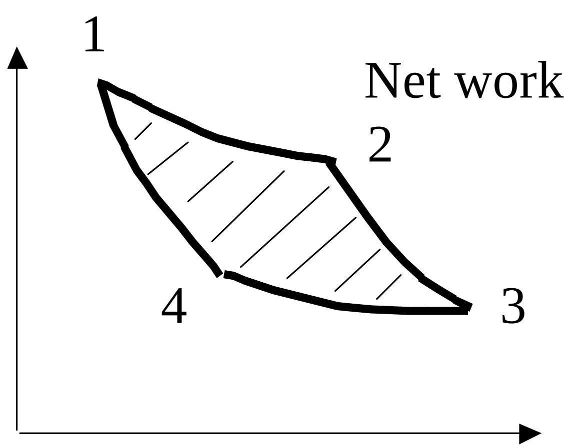


Work done on gas = ∫PdV, area under the process curve 3-4-1

subtract

Since dV<0
∫PdV<0





The Carnot Principles

- The efficiency of an irreversible heat engine is always less than the efficiency of a reversible one operating between the same two reservoirs. $\eta_{th,\ irrev} < \eta_{th,\ rev}$

- The efficiencies of all reversible heat engines operating between the same two reservoirs are the same. $(\eta_{th,\ rev})_A = (\eta_{th,\ rev})_B$

- Both Can be demonstrated using the second law (K-P statement and C-statement). Therefore, the Carnot heat engine defines the maximum efficiency any practical heat engine can reach up to.

- Thermal efficiency $\eta_{th}=W_{net}/Q_H=1-(Q_L/Q_H)=f(T_L,T_H)$ and it can be shown that $\eta_{th}=1-(Q_L/Q_H)=1-(T_L/T_H)$. This is called the Carnot efficiency.

- For a typical steam power plant operating between T_H=800 K (boiler) and T_L=300 K(cooling tower), the maximum achievable efficiency is 62.5%.

Example

Let us analyze an ideal gas undergoing a Carnot cycle between two temperatures T_H and T_L.

- 1 to 2, isothermal expansion, $\Delta U_{12} = 0$

$Q_H = Q_{12} = W_{12} = \int PdV = mRT_H \ln(V_2/V_1)$

- 2 to 3, adiabatic expansion, $Q_{23} = 0$

$(T_L/T_H) = (V_2/V_3)^{k-1} \rightarrow (1)$

- 3 to 4, isothermal compression, $\Delta U_{34} = 0$

$Q_L = Q_{34} = W_{34} = - mRT_L \ln(V_4/V_3)$

- 4 to 1, adiabatic compression, $Q_{41} = 0$

$(T_L/T_H) = (V_1/V_4)^{k-1} \rightarrow (2)$

From (1) & (2), $(V_2/V_3) = (V_1/V_4)$ and $(V_2/V_1) = (V_3/V_4)$

$\eta_{th} = 1-(Q_L/Q_H) = 1-(T_L/T_H)$ since $\ln(V_2/V_1) = \ln(V_4/V_3)$

It has been proven that $\eta_{th} = 1-(Q_L/Q_H) = 1-(T_L/T_H)$ for all Carnot engines since the Carnot efficiency is independent of the working substance.

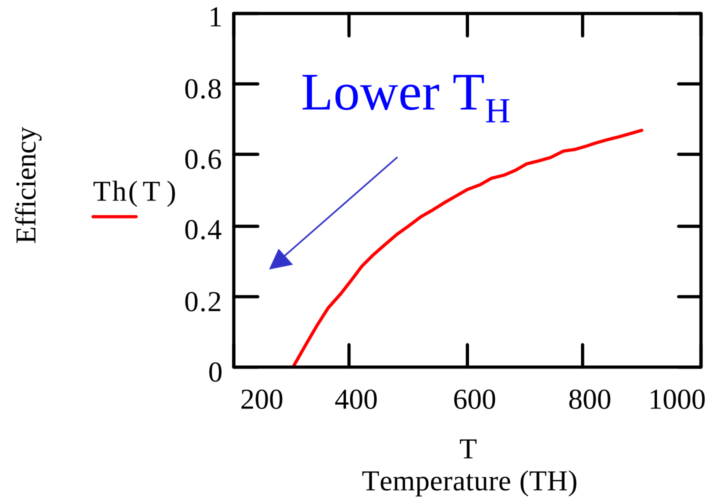
Carnot Efficiency

A Carnot heat engine operating between a high-temperature source at 900 K and reject heat to a low-temperature reservoir at 300 K. (a) Determine the thermal efficiency of the engine. (b) If the temperature of the high-temperature source is decreased incrementally, how is the thermal efficiency changes with the temperature.

$$\eta_{th} = 1 - \frac{T_L}{T_H} = 1 - \frac{300}{900} = 0.667 = 66.7\%$$

Fixed $\mathrm{T_L} = 300(K)$ and lowering $\mathrm{T_H}$

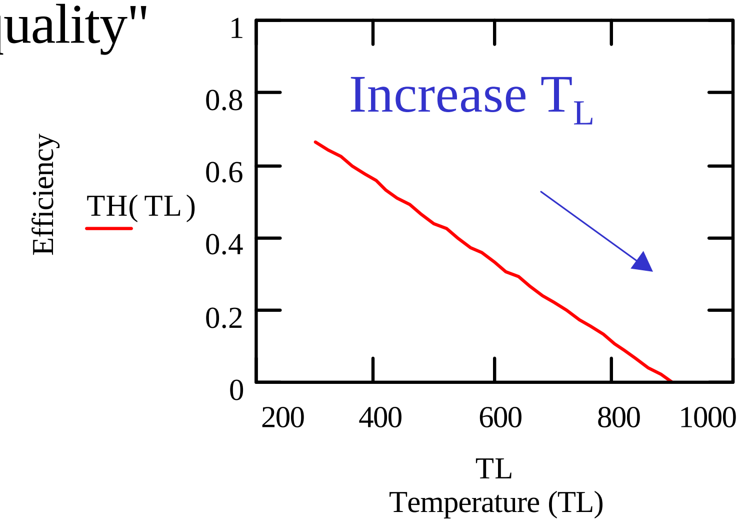
$$\eta_{th}(T_H) = 1 - \frac{300}{T_H}$$



The higher the temperature, the higher the "quality" of the energy: More work can be done

Fixed $\mathrm{T_H} = 900(K)$ and increasing $\mathrm{T_L}$

$$\eta_{th}(T_H) = 1 - \frac{T_L}{900}$$



Carnot Efficiency

• Similarly, the higher the temperature of the low-temperature sink, the more difficult for a heat engine to transfer heat into it, thus, lower thermal efficiency also. That is why low-temperature reservoirs such as rivers and lakes are popular for this reason.

•To increase the thermal efficiency of a gas power turbine, one would like to increase the temperature of the combustion chamber. However, that sometimes conflict with other design requirements. Example: turbine blades can not withstand the high temperature gas, thus leads to early fatigue. Solutions: better material research and/or innovative cooling design.

• Work is in general more valuable compared to heat since the work can convert to heat almost 100% but not the other way around. Heat becomes useless when it is transferred to a low-temperature source because the thermal efficiency will be very low according to $\eta_{th}=1-(T_L/T_H)$. This is why there is little incentive to extract the massive thermal energy stored in the oceans and lakes.