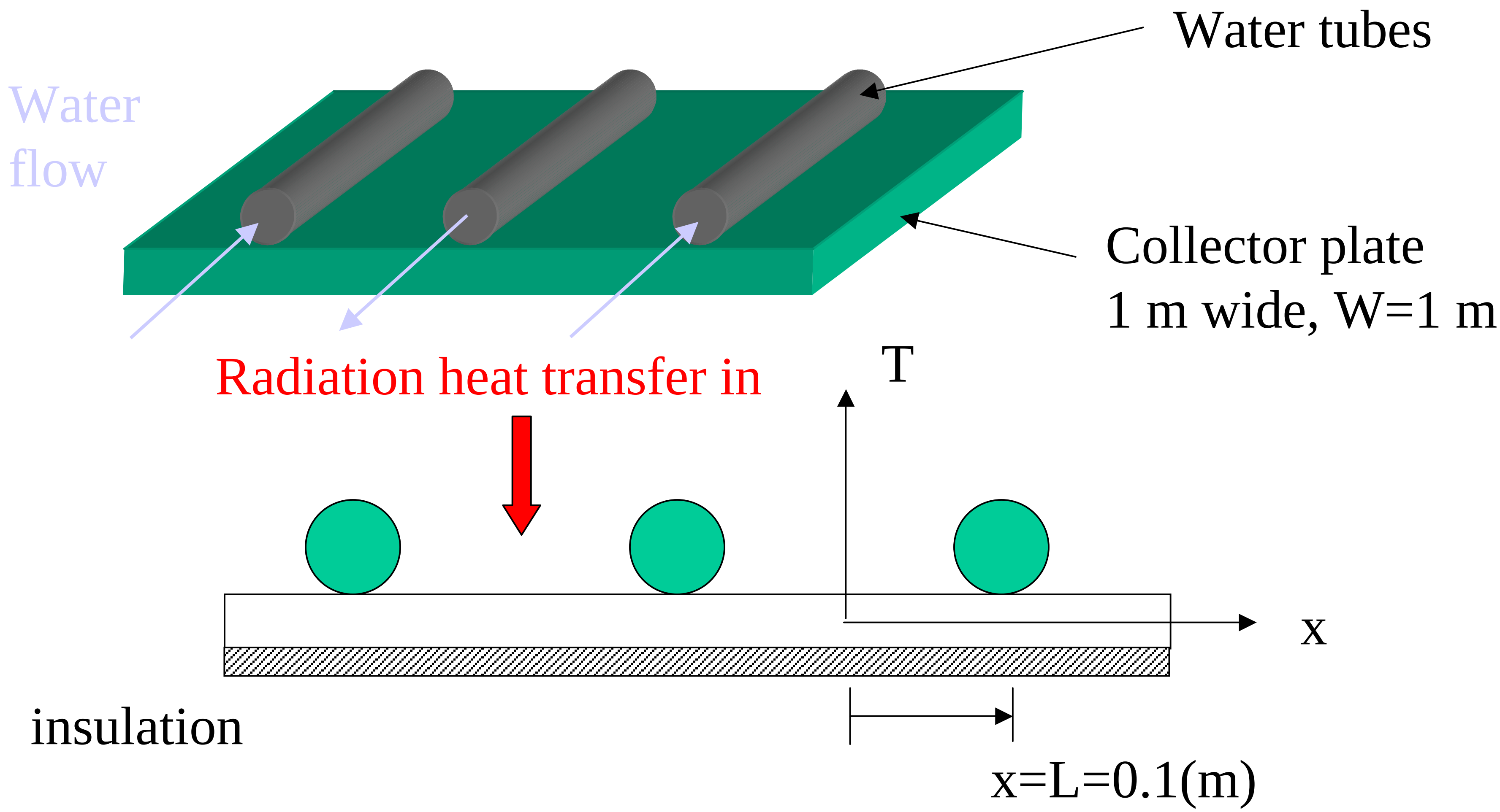
Solar Collector Plate



Question: what will be the temperature distribution of the plate, $T(x)$, if there is a constant radiation heat flux q" coming in. For simplicity, assume the temperature immediately below the water tube is constant, $T(x=L)=T_O$.

Temperature Distribution

Model the solar heat flux as continuous generation inside the plate:

averge solar flux around noon time: $q'' = 700\ \mathrm{W/m^2}$,

Assume it uniformly distributes inside the plate with thickness $\delta = 1$ mm

$\dot{q} = q''/\delta = 700000\ \mathrm{W/m^3}$, separation between tubes 2L = 2 cm, L = 10 cm

Heat diffusion equation: $\nabla^2 T + \dfrac{\dot{q}}{k} = 0$, $k = 240$ W / m K for Aluminum

with boundary conditions: $\dfrac{dT}{dx}(x=0) = 0$, $T(x=L) = T_O = 40°C$

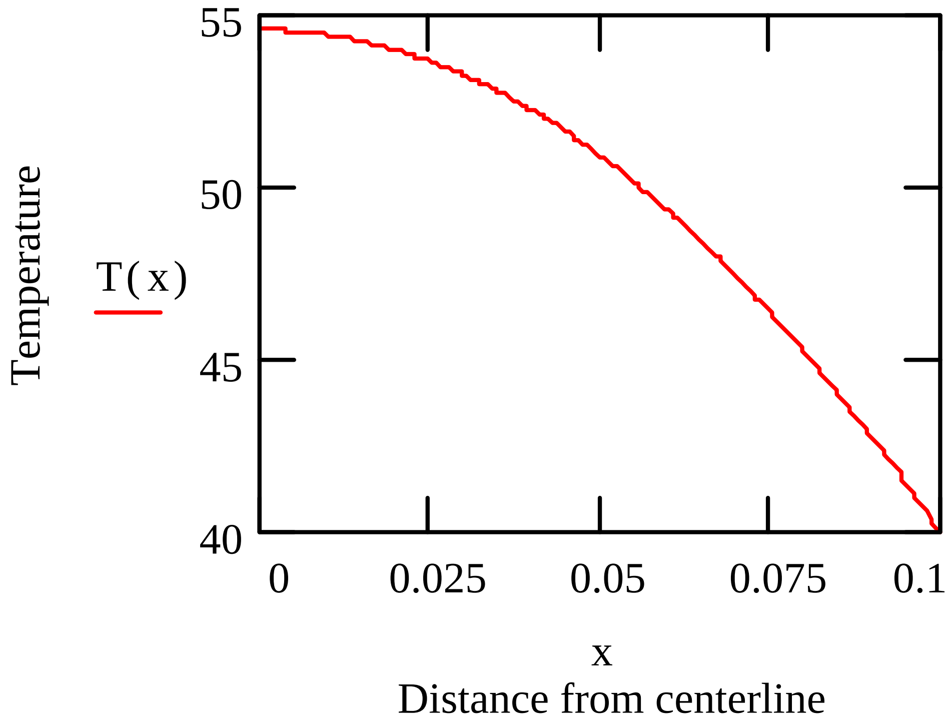
$$\frac{d^2T}{dx^2} = -\frac{\dot{q}}{k},\ \text{integrate twice to get } T(x) = -\frac{\dot{q}}{2k}x^2 + Ax + B$$

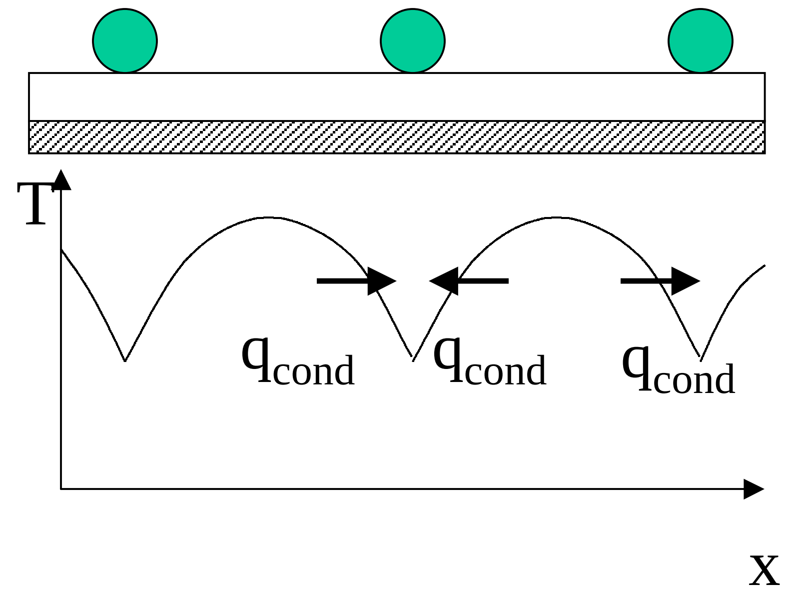
Apply boundary conditions: $A = 0$, $B = 40 + \dfrac{\dot{q}}{2k}L^2$

$$T(x) = 40 + \frac{\dot{q}}{2k}(L^2 - x^2) = 40 + 1458.3(0.01 - x^2)$$

Temperature Distribution on the Plate

$$T(x) = 40 + \frac{\dot{q}}{2k}(L^2 - x^2) = 40 + 1458.3(0.01 - x^2)$$





$$q''_{cond} = -k\frac{dT}{dx} = \dot{q}x$$

Heat Transfer to the Tube

$$q''_{cond} = -k\frac{dT}{dx} = \dot{q}x,$$

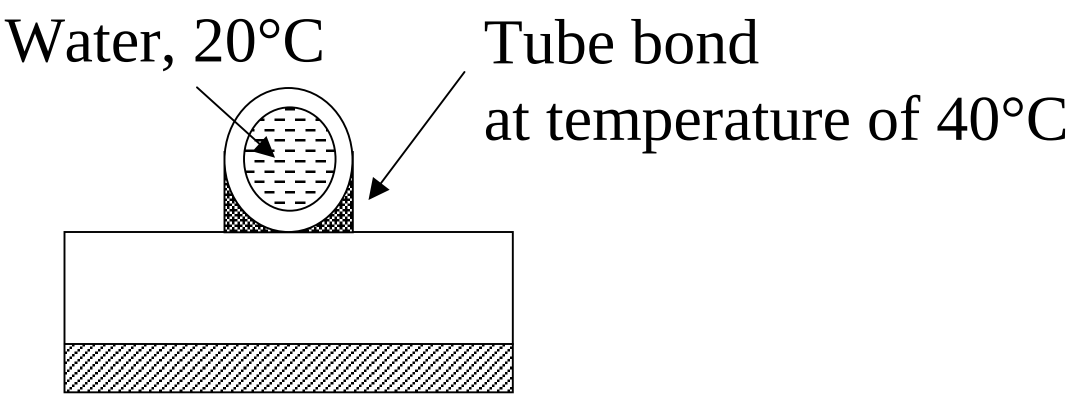
Total heat transfer $q(x) = q''_{cond}\, A = q''_{cond}\, (W)(\delta) = \dot{q}(W)(\delta)(x)$

$q(x = 0) = 0$, no heat transfer at the middle between two tubes

$q(x = L) = \dot{q}(W)(\delta)(L) = 700000(1)(0.001)(0.1) = 70(W)$

The total heat transfer into the tube should be twice of this amount

$$q_{tube} = 2q(x = L) = 140(W)$$



$q_{tube} = hA_s(T-T_\infty)$

$A_s = \pi RW$

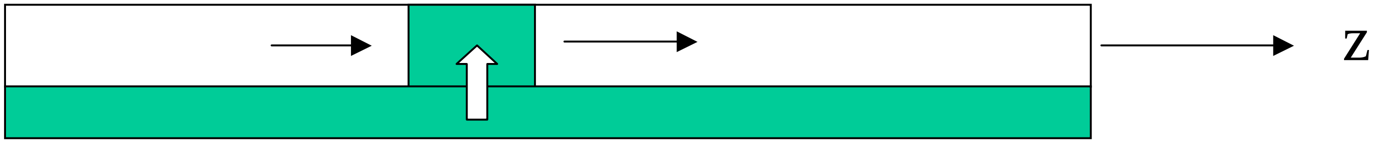
$h = 140/[(\pi RW)(T- T_\infty)]$

$= 111.5(W/m^2K)$

for a 2-cm radius pipe and an incoming water temperature of 20°C

Temperature Distribution in Flow Direction

$$\dot{E}_z = \dot{m}c_p T_z \qquad \dot{E}_{z+dz} = \dot{E}_z + \frac{d\dot{E}_z}{dz}dz = \dot{m}c_p T_z + \dot{m}c_p \frac{dT_z}{dz}dz$$



Assume constant heat flux q” into the tube

$$0 = q_{in} - q_{out} + \dot{E}_z - \dot{E}_{z+dz}$$

$$0 = q''(dzR) - \frac{d\dot{E}_z}{dz}dz = q''(dzR) - \dot{m}c_p dT_z$$

$$\frac{dT_z}{dz} = \frac{q''R}{\dot{m}c_p},$$ integrate from z = 0 to z = W

$$\mathrm{T}_z(z) = T_z(0) + \frac{q''R}{\dot{m}c_p}z,$$ the temperature increases linearly

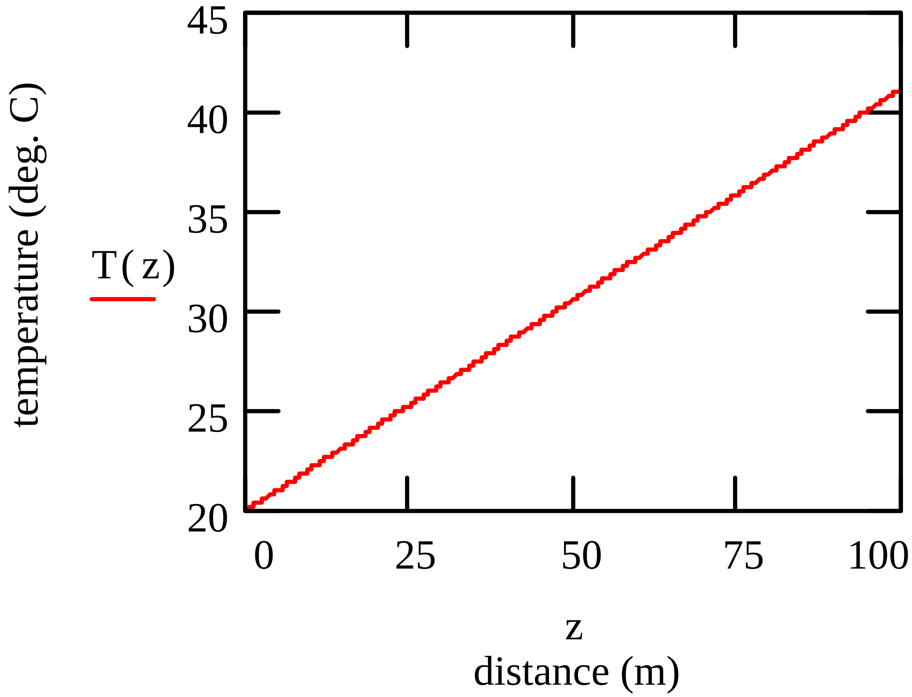
due to the heat transfer from the collector plate

Temperature Distribution in Flow Direction (2)

$$T_z(z) = T_z(0) + \frac{q''R}{\dot{m}c_p} z,$$

$q'' = h(T_O - T_\infty) = 2230$ (W/m^2), $R = 0.02$ (m), $dm/dt = 0.05$ (kg/s), $c_p = 4200$ (J/kg K)

$T_z(z) = T_z(0) + 0.212z = 20 + 0.212z$



- Temperature increases linearly. It increases 0.212°C for every 1 m increase in length. The temperature increase in water can be significant if the tube length is long.

Temperature distribution in flow direction (3)

Assume h(convection heat transfer coefficient) is constant; q” is not a constant)

$$0 = q_{in} - q_{out} + \dot{E}_z - \dot{E}_{z+dz}$$

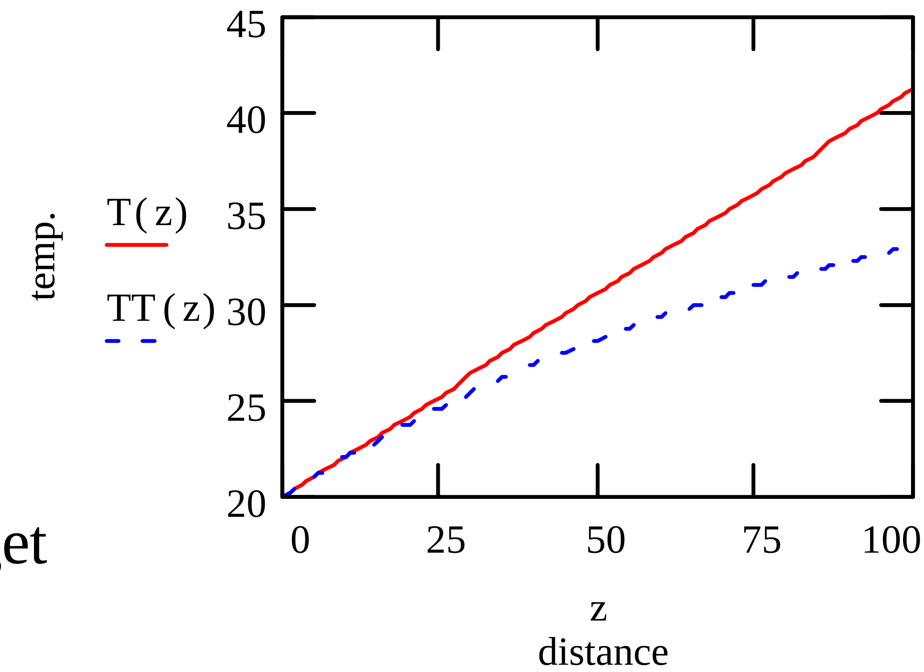
$$0 = q''(dzR) - \frac{d\dot{E}_z}{dz}dz$$

$$= h(dzR)(T_O - T_z) - \dot{m}c_p dT_z$$

$$\frac{dT_z}{(T_O - T_z)} = \frac{hRdz}{\dot{m}c_p}, \text{ integrate to get}$$

$$\mathrm{T}_z(z) = T_O + (T_\infty - T_O)\exp\left(-\frac{hR}{\dot{m}c_p}z\right)$$

$$= 40 - 20\exp(-0.0106z)$$



As the water temperature increases, both the heat transfer and the rate of temperature change decrease also.