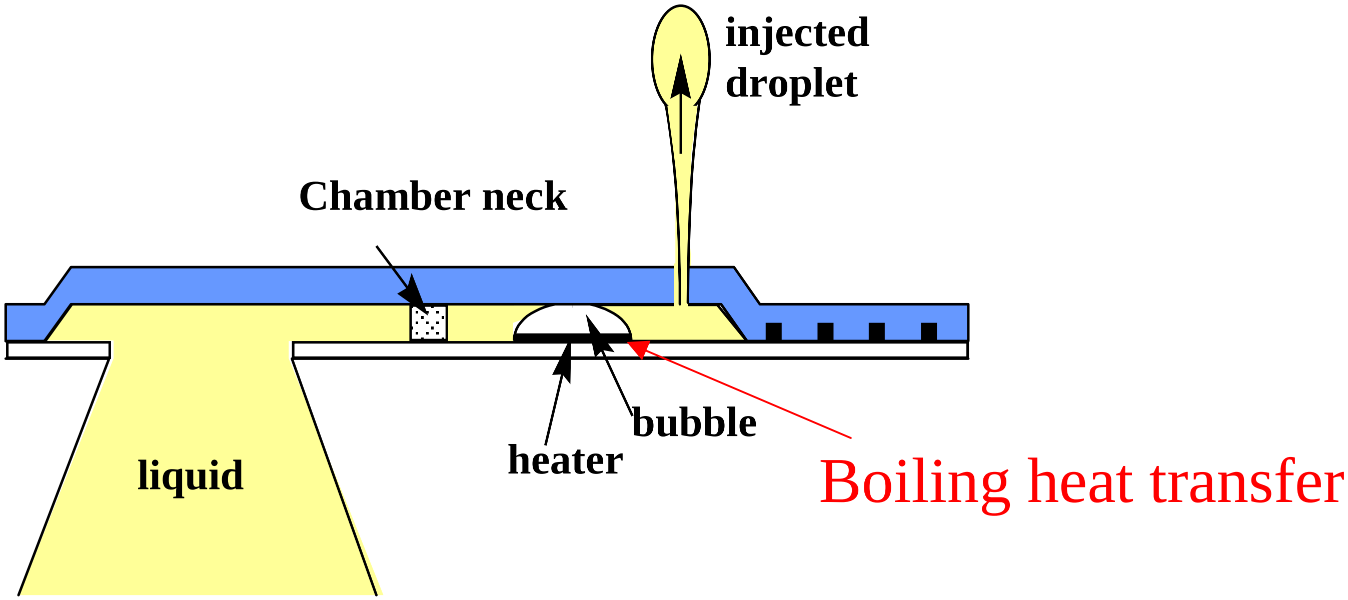
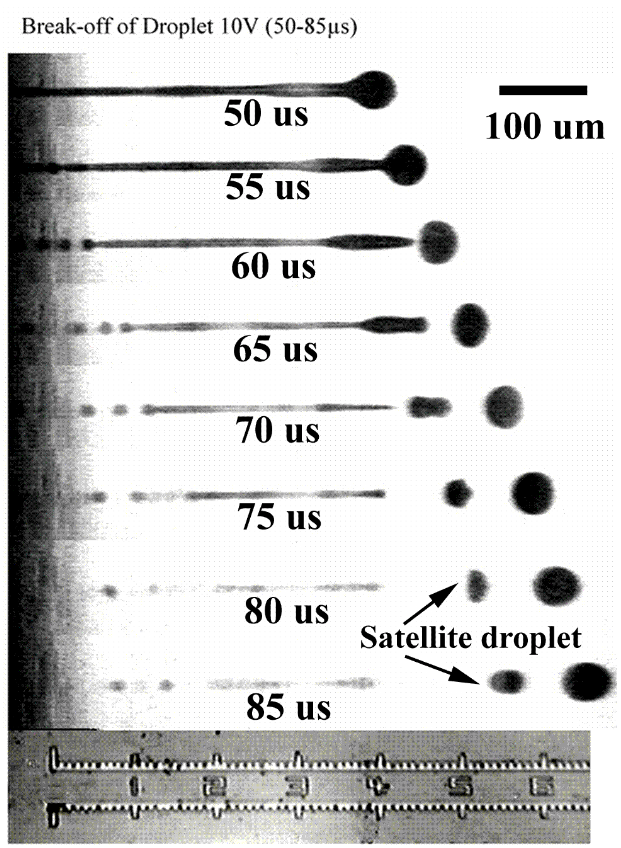
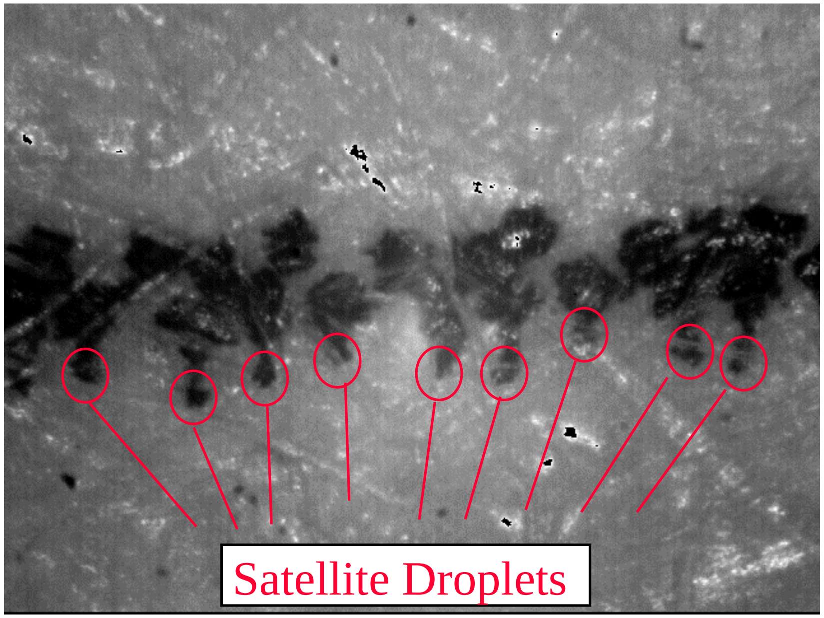
Operational Principle of Thermal Bubble Jet



- Current pulse heats up liquid
- Bubble as a pump pushes out droplet
- Refill by capillary force

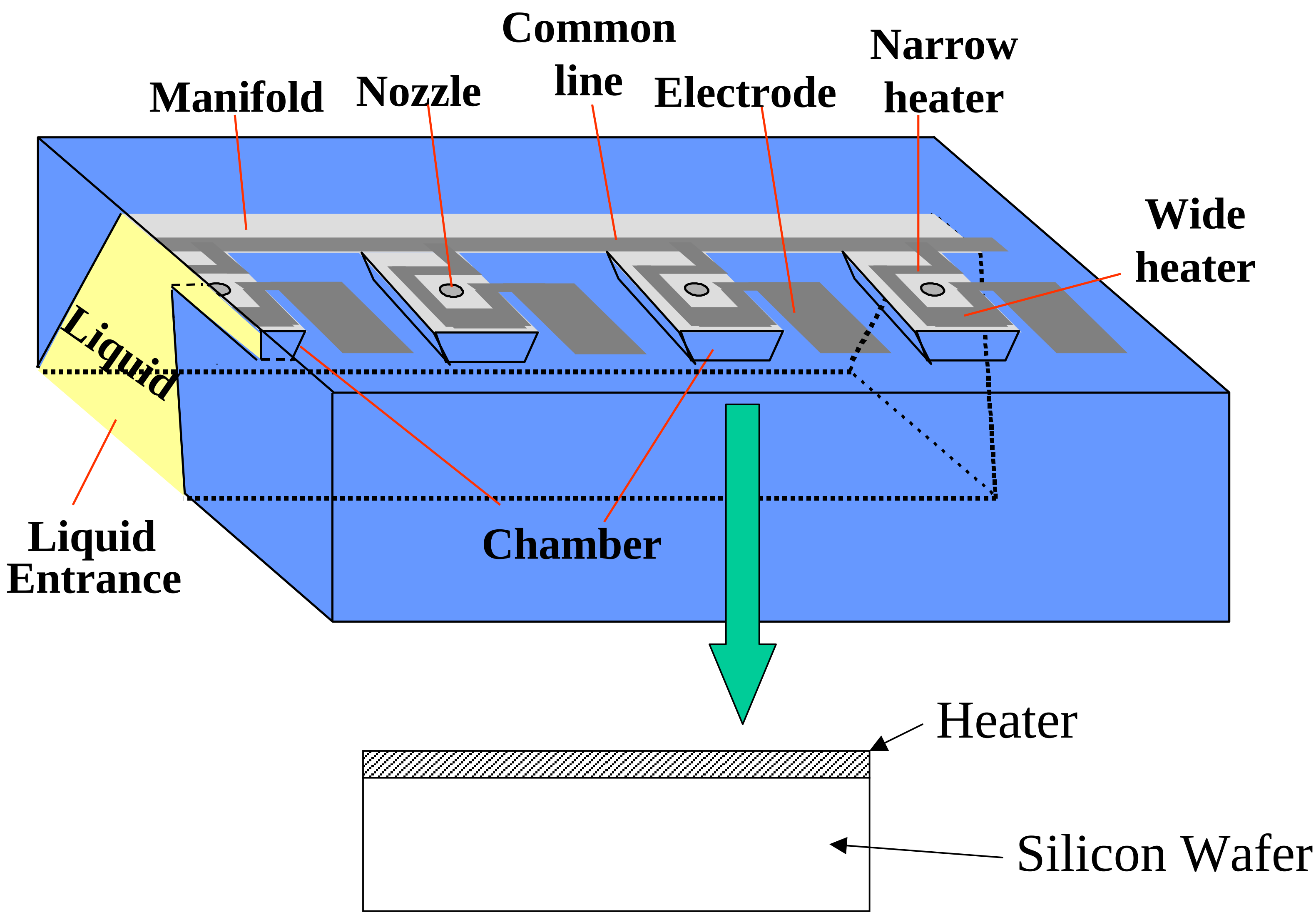
Satellite Droplets



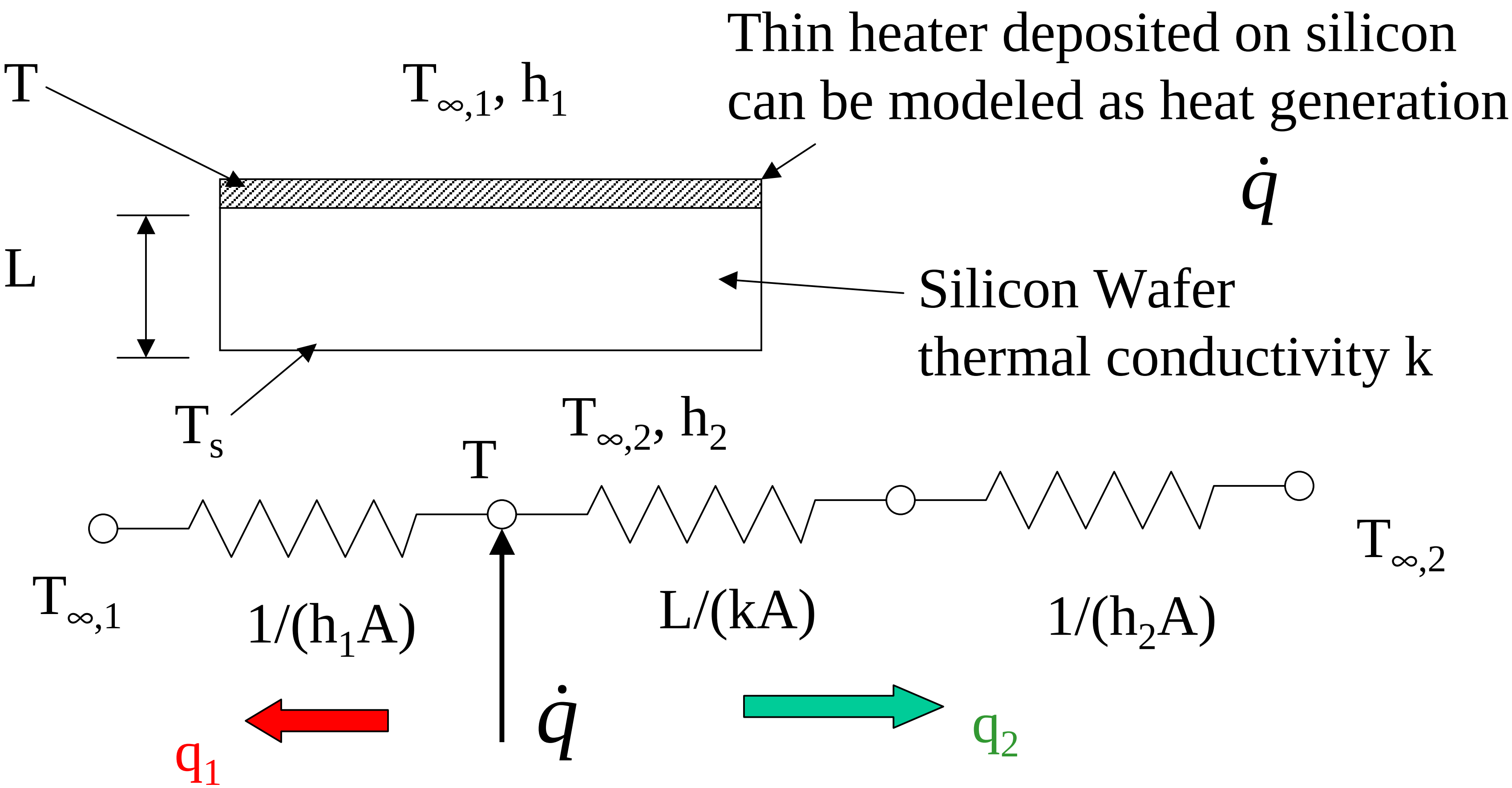


Inkjet Droplet injection sequence

Micro-Machined Thermal Bubble Jet

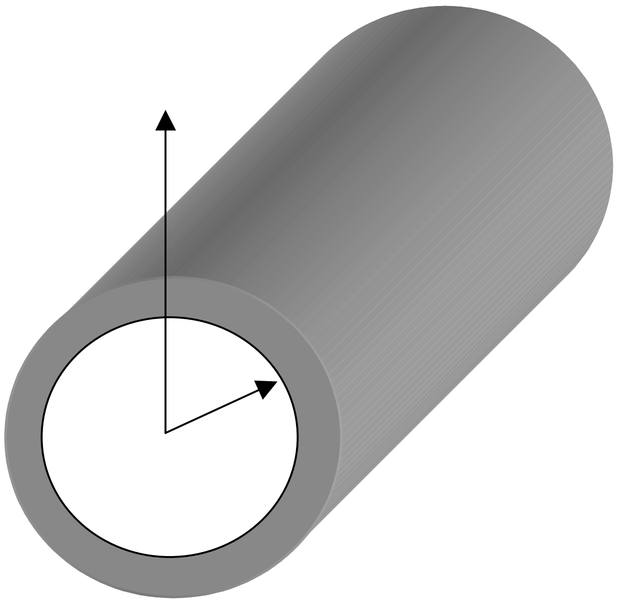


1-D Heat Conduction with Generation



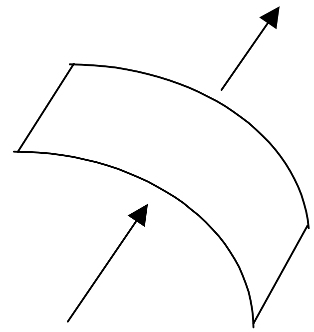
$$\dot{q}\forall = i^2 R = q_1 + q_2 = \frac{T - T_{\infty,1}}{1/(h_1 A)} + \frac{T - T_{\infty,2}}{L/(kA) + 1/(h_2 A)}$$

1-D, steady Heat Transfer in Cylindrical Coordinate



Radial direction

$q_{r+dr}=qr+(dq_r/dr)dr$



$q_r=-kA_r(dT/dr)$

Net heat transfer: $q_{\mathrm{r}} - q_{r+dr} = 0$ since no generation, steady state

$$-\frac{\mathrm{d}}{\mathrm{dr}}\left(-kA\frac{dT}{dr}\right) = -\frac{d}{dr}\left[-k(2\pi rL)\frac{dT}{dr}\right] = 0, \text{ assume k = constant}$$

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0, \text{ integrate twice and apply boundary conditions}$$

Cylindrical Heat Conduction

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0, \text{ integrate once: } \mathrm{r}\frac{dT}{dr} = C_1, \ \frac{dT}{dr} = \frac{C_1}{r}$$

Integrate again: $T = C_1 \ln r + C_2$

Apply boundary conditions, $\mathrm{T}(\mathrm{r}_1) = T_1$ and $\mathrm{T}(\mathrm{r}_2) = T_2$

$$C_1 = \frac{T_1 - T_2}{\ln(r_1 / r_2)}, \ C_2 = T_2 - \frac{T_1 - T_2}{\ln(r_1 / r_2)}\ln(r_2)$$

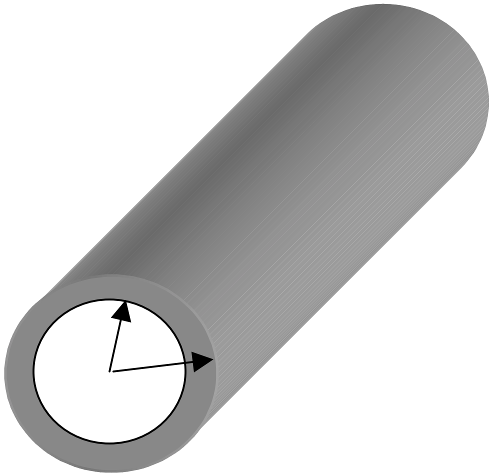
$$T(r) = \frac{T_1 - T_2}{\ln(r_1 / r_2)}\ln(r / r_2) + T_2,$$

temperature distribution along the radial direction

$$q = -kA\frac{dT}{dr} = \frac{2\pi kL(T_1 - T_2)}{\ln(r_2 / r_1)} = \frac{(T_1 - T_2)}{R_{th}}$$

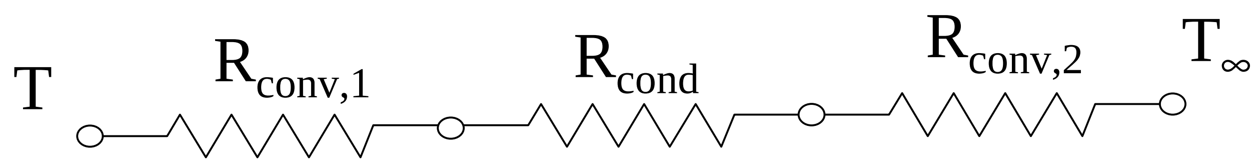
Thermal resistance in cylindrical coordinate $\mathrm{R}_{\mathrm{th}} = \dfrac{\ln(r_2 / r_1)}{2\pi kL}$

Heat Loss from a Cylindrical Pipe (no insulation)



Steam at 300°C flow in a cast iron circular pipe (k=80W/m.K). Determine the heat loss per unit length of the pipe to the surroundings at T_∞=20°C, with a combined (radiation & convection) heat transfer coefficient of h_2=20 W/m^2.K). The convection coefficient inside the pipe is h_1=50 W/m^2.K)

r_1=1 cm
r_2=1.25 cm



$$q = \frac{T - T_\infty}{\sum R_{th}} = \frac{T - T_\infty}{R_{conv,1} + R_{cond} + R_{conv,2}} = \frac{T - T_\infty}{\frac{1}{h_1 A} + \frac{\ln(r_2 / r_1)}{2\pi k L} + \frac{1}{h_2 A}}$$

Heat Loss from a Cylindrical Pipe (no insulation) (cont.)

$$R_{conv,1} = \frac{1}{h_1 A_1} = \frac{1}{(50)(2\pi r_1)(1)} = 0.318(K/W)$$

$$R_{cond} = \frac{\ln(r_2/r_1)}{2\pi k L} = \frac{\ln(1.25/1)}{2\pi(80)(1)} = 0.000807(K/W)$$

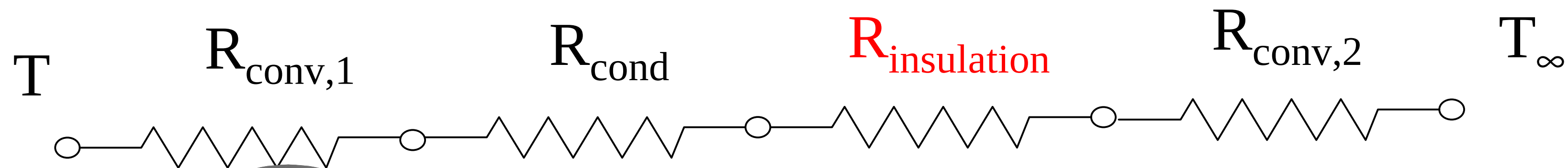
$$R_{conv,2} = \frac{1}{h_2 A_2} = \frac{1}{(20)(2\pi r_2)(1)} = 0.637(K/W)$$

$$q = \frac{T - T_\infty}{\sum R_{th}} = \frac{300 - 20}{0.318 + 0.000807 + 0.637} = 292.9(W)$$

Note: the cast iron pipe is a good conductor, therefore, it has a small thermal resistance. To prevent heat loss, it is recommended that insulation be used outside the pipe.

Insulation

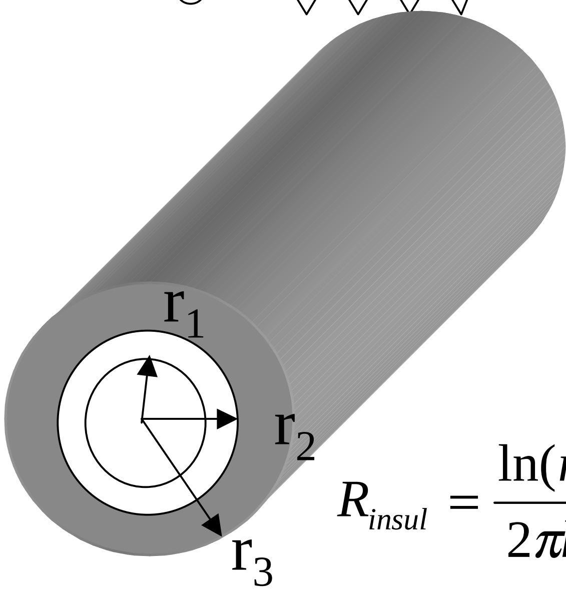
It is suggested that a thick layer of insulation (k=0.5 W/m.K) be wrapped outside the pipe. Determine the heat loss as a function of the thickness of the fiber glass wrap.



Assume the fiber thickness is t

$r_3=r_2+t$

$R_{conv,1}$ and R_{cond} stay the same, unchanged



$$R_{insul}=\frac{\ln(r_3/r_2)}{2\pi k_{insul}L}=\frac{\ln(1+t/r_2)}{2\pi(0.5)(1)}=0.318*\ln(1+80t)$$

r_1=1 cm

r_2=1.25 cm

$$R_{conv,2}=\frac{1}{h_2A_3}=\frac{1}{h_2(2\pi r_3L)}=\frac{1}{(20)(2\pi)(0.0125+t)(1)}=\frac{1}{125.6(0.0125+t)}$$

Insulation-2

$$q = \frac{T - T_\infty}{\sum R_{th}} = \frac{T - T_\infty}{R_{conv,1} + R_{cond} + R_{insul} + R_{conv,2}}$$

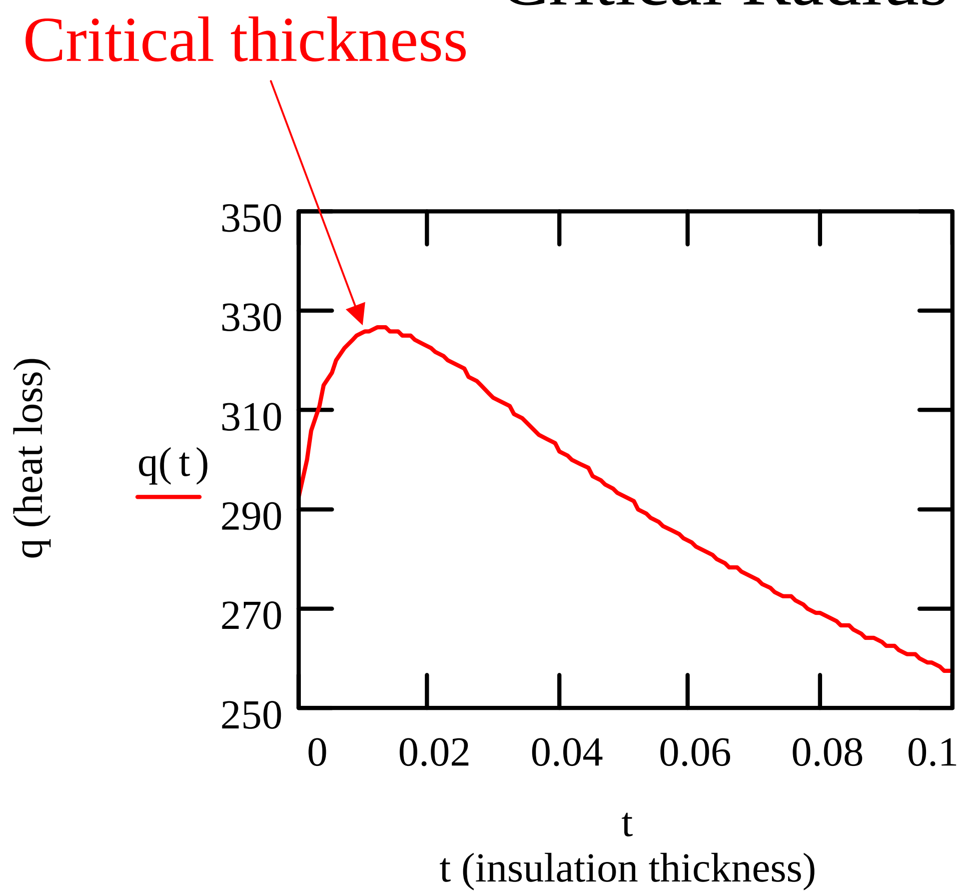
$$= \frac{T - T_\infty}{\frac{1}{h_1 A} + \frac{\ln(r_2 / r_1)}{2\pi k L} + \frac{\ln(r_3 / r_2)}{2\pi k_{insul} L} + \frac{1}{h_2 A_3}}$$

$$= \frac{300 - 20}{0.318 + 0.000807 + 0.318 \ln(1 + 80t) + \frac{1}{125.6(0.0125 + t)}}$$

$$= \frac{280}{0.319 + 0.318 \ln(1 + 80t) + \frac{1}{125.6(0.0125 + t)}}$$

- Plot this function in the following slide

Critical Radius of Insulation



- Heat loss increases initially when one adds insulation (why?)
- Maximum heat loss at critical radius of insulation that $r_{critical} = k_{insul}/h_{outside}$
- In the present case, $r_{critical}=0.5/20=0.025$
- Its critical thickness is $t=0.025-0.0125=0.0125(m)$

Note: similar derivation can be made for a spherical container. The critical radius for a spherical shell is $r_{critical}=2k/h$