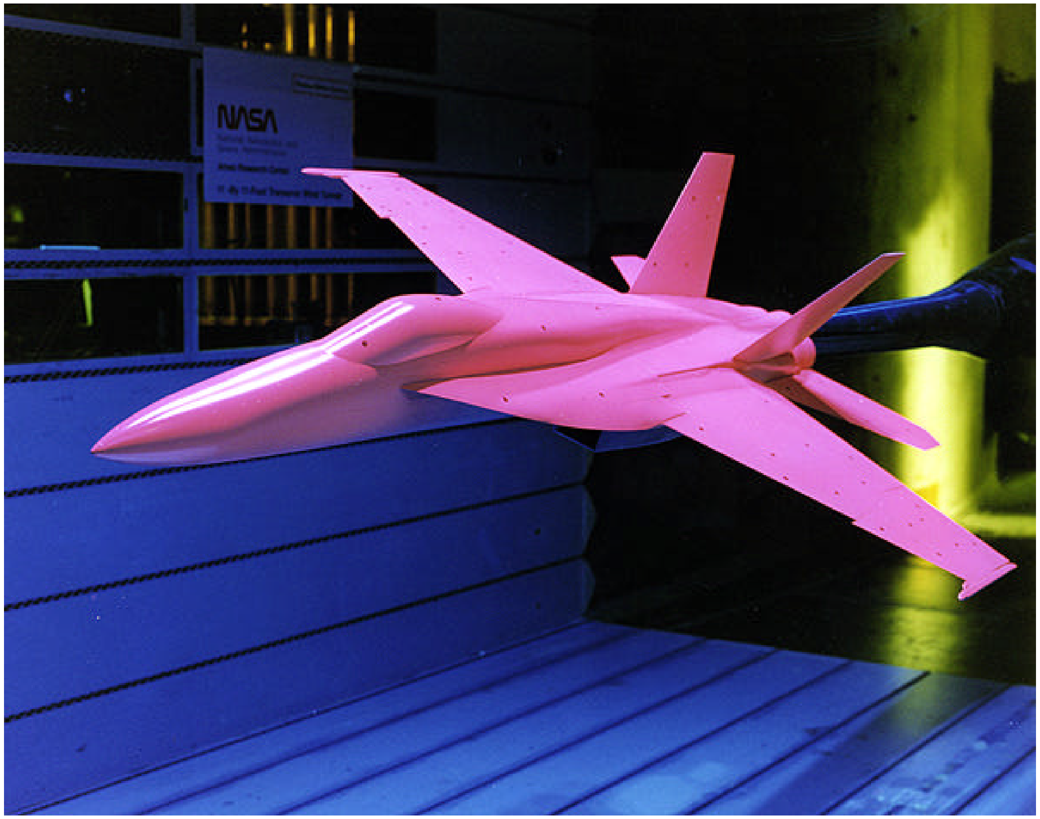
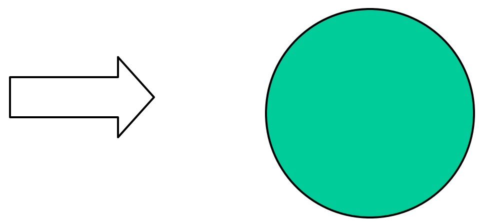
Flow Similarity and Model Testing

- Geometrical similarity: similar geometry.
- Kinematic similarity: flow pattern around the model should be similar to...
- Dynamical similarity: forces acting on the model should be similar to the real object to be analyzed.



Flight test of a F-18 model in NASA 11 ft transonic (Mach .4-1.5) wind tunnel

Aerodynamic forces on a Bluff Body (a Sphere)



Relevant parameters:

- free-stream velocity, U
- diameter of the sphere, D
- density of the fluid, ρ
- viscosity of the fluid, μ
- forces on the sphere, F_L & F_D

$F_L = 0$ because of symmetry

$F_D = f(U,D,\rho,\mu)$

To carry out a set of experiments to characterize the drag forces on spheres, one need to vary four parameters independently: U,D,ρ,μ.

- It is very time consuming and the resulting data set will be difficult to analyze.
- Some kinds of scaling are needed to apply the wind tunnel testing data to a real flow problem.

Dimensional Analysis

• Drag force (F_D) depends on the pressure/shear stress distributions and should scale to the flow kinetic energy: (1/2)ρU^2 and the surface area of the model: D^2.

Define drag coefficient: $$C_D = \frac{F_D}{\frac{1}{2} r U^2 D^2} = \frac{f(U, D, r, m)}{\frac{1}{2} r U^2 D^2} = g(U, D, r, m)$$

• The drag coefficient is a dimensionless variable, therefore, the new function g must be consisting of dimensionless variables only. Otherwise, a choice of different unit systems will provide different results for the same system.

• It can be shown, by using the **Buckingham Pi theorem**, that the new function consists of one dimensionless variable only, that is, the Reynolds number, Re.

$$\mathrm{Re} = \frac{rUD}{m}.$$

That is the drag coefficient is the function of one parameter only:

$$C_D = g(\mathrm{Re}) = g\left(\frac{rUD}{m}\right)$$

See golf ball aerodynamics

Common Dimensionless Groups in Fluid Mechanics

$\frac{rVL}{m}$	Reynolds number, Re	Inertia force / viscous force	Always important
$\frac{V}{c}$	Mach number, M	Inertia force / compressibility force	Compressible flow
$\frac{V}{\sqrt{gL}}$	Froude number, Fr	Inertia force / gravitational force	Free surface flow
$\frac{wL}{V}$	Strouhal number, St	Unsteady force / convective force	Unsteady flow

- c: speed of the sound
- ω: frequency of the oscillatory flow
- L: characteristic length
- V: characteristic velocity