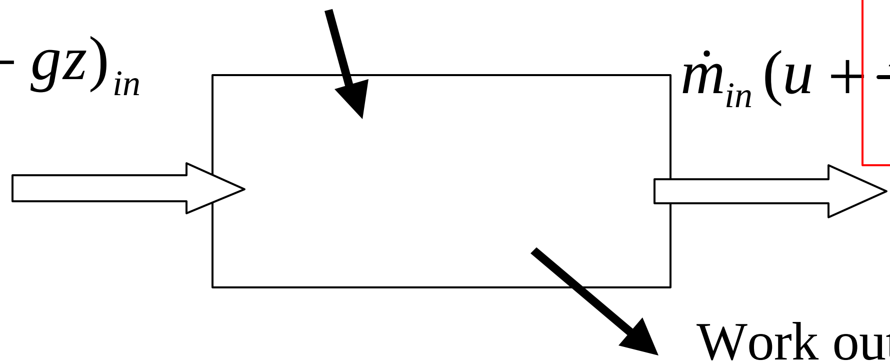
Steady Energy Equation

$$\dot{m}_{in}(u+\frac{p}{r}+\frac{V^2}{2}+gz)_{in}$$

Heat in q=dQ/dt



$$\dot{m}_{in}(u+\frac{p}{r}+\frac{V^2}{2}+gz)_{out}$$

Pressure flow work

Work out dW/dt

From mass conservation: $\dot{m}_{in}=\dot{m}_{out}=\dot{m}$

From the First law of Thermodynamics (Energy Conservation):

$$\frac{\mathrm{dQ}}{\mathrm{dt}}+\dot{m}(u+\frac{p}{r}+\frac{V^2}{2}+gz)_{in}=\dot{m}(u+\frac{p}{r}+\frac{V^2}{2}+gz)_{out}+\frac{dW}{dt},\ \text{or}$$

$$\frac{\mathrm{dQ}}{\mathrm{dt}}+\dot{m}(h+\frac{V^2}{2}+gz)_{in}=\dot{m}(h+\frac{V^2}{2}+gz)_{out}+\frac{dW}{dt}$$

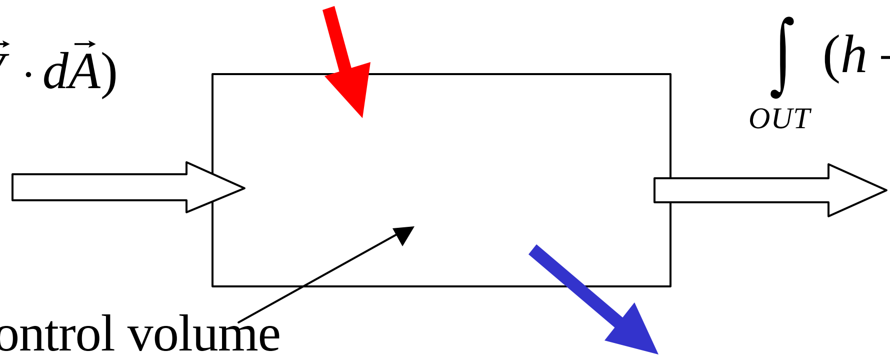
where $h=u+\frac{p}{r}$ is defined as "enthaply"

Generalized Energy Equation

Heat in q=dQ/dt

$$\int_{IN} (h + \frac{V^2}{2} + gz)(r\vec{V} \cdot d\vec{A})$$

$$\int_{OUT} (h + \frac{V^2}{2} + gz)(r\vec{V} \cdot d\vec{A})$$



Energy inside the control volume

$$\int_{CV} r(\mathrm{u} + \frac{\mathrm{V}^2}{2} + gz)d\forall$$

Work out dW/dt,
excluding the pressure flow work

Energy balance:

[Time rate change of energy inside the control volume]

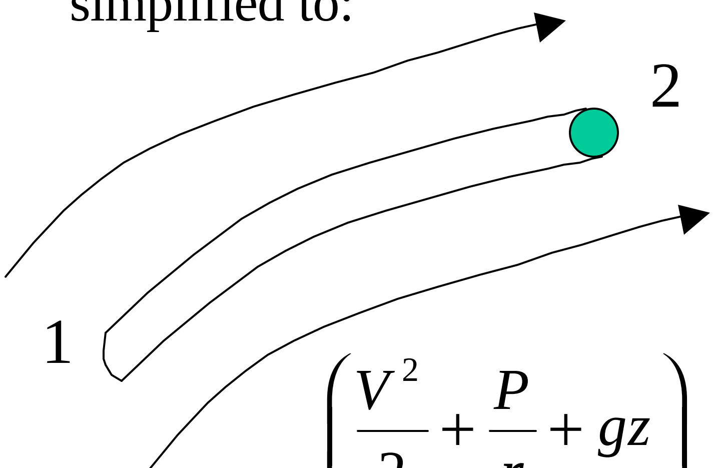
+[energy flow in and out of the control surfaces]

=[external heat transfer]-[external work]

$$\frac{\partial}{\partial t}\int_{CV} r(\mathrm{u} + \frac{\mathrm{V}^2}{2} + gz)d\forall + \int_{CS} (h + \frac{V^2}{2} + gz)(r\vec{V} \cdot d\vec{A}) = \frac{dQ}{dt} - \frac{dW}{dt}$$

Bernoulli Equation

Consider the control volume is placed around a streamtube inside a steady, incompressible, isothermal and inviscid (no viscosity) flow as shown below. No external heat transfer and no external works. The energy equation can be simplified to:



$$\left(\frac{V^2}{2}+\frac{P}{r}+gz\right)_{IN}=\left(\frac{V^2}{2}+\frac{P}{r}+gz\right)_{OUT}$$

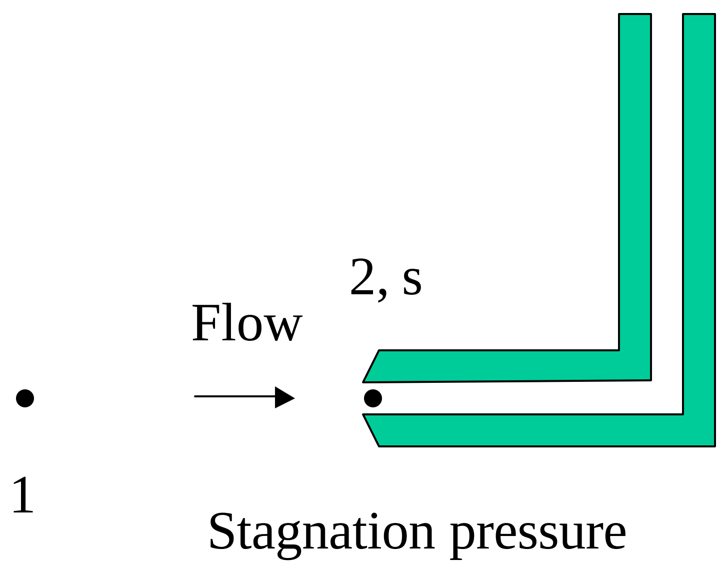
internal energy is a constant since the temperature is a constant

All flow properties are constant across a streamtube since the cross section of the tube is very small

$$\frac{V_1^2}{2}+\frac{P_1}{r}+gz_1=\frac{V_2^2}{2}+\frac{P_2}{r}+gz_2, \text{ or } \frac{V^2}{2}+\frac{P}{r}+gz=\text{constant}$$

Bernoulli Equation (Example)

The pitot tube is a simple device to be used for airspeed measurement. It can usually be seen close to the nose of the airplane. The basic operation of the tube is based on the Bernoulli's principle. One end of the tube is directly pointing toward the incoming air stream. As the air particles approach the opening, they will gradually be decelerated and eventually come to a complete stop, or stagnant. During the process, the kinetic energy of the particles convert into higher pressure, stagnation pressure. The difference of this pressure and the local static pressure can be used to determine the local air speed.



Along the streamline from 1 to 2,

The Bernoulii equation is valid:

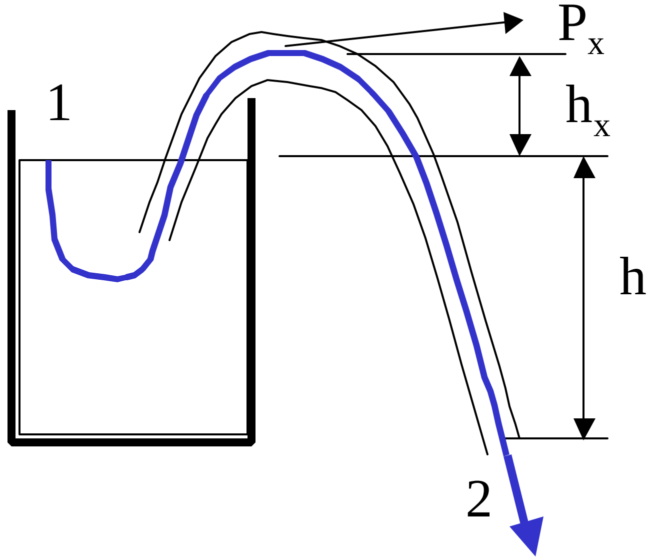
$$\frac{P_1}{r} + \frac{V_1^2}{2} = \frac{P_2}{r} + \frac{V_2^2}{2} = \frac{P_{stag}}{r} \text{ (since } V_2 = 0)$$

$$V_1 = \sqrt{\frac{2(P_{stag} - P_1)}{r}}$$

Therefore, the measurement of $P_{stag} - P_1$ can determine the local air speed.

Siphon Example

Water is siphoned out from an aquarium tank using a tube as shown. Determine the draining flow rate as a function of the height difference (h) between the free surface and the tube outlet.



Apply Bernoulli equation between two points 1 & 2 along a streamline:

$$\frac{P_1}{r} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{r} + \frac{V_2^2}{2} + gz_2$$

$P_1 = P_2 = P_{atm}$ and $V_1 << V_2$

$$V_2 = \sqrt{2g(z_1 - z_2)} = \sqrt{2gh}$$

Determine the pressure P_x at a point x inside the tube as shown

$$\frac{P_1}{r} + \frac{V_1^2}{2} + gz_1 = \frac{P_x}{r} + \frac{V_x^2}{2} + gz_x,\ V_x = V_2 = \sqrt{2gh},\ V_1 << V_x$$

$$P_x = P_1 + r\,g(z_1 - z_x) - \frac{rV_x^2}{2} = P_1 + r\,g(z_1 - z_x) - r\,gh = P_1 + r\,g[(z_1 - z_x) - h]$$

$= P_1 - r\,g(h_x + h)$, Pressure can be very low $\Rightarrow$ cavitation.