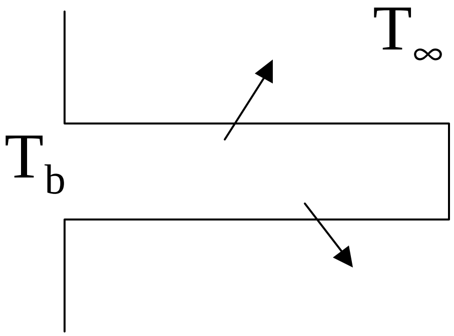
Fin Design



Total heat loss: q_f=Mtanh(mL) for an adiabatic fin, or q_f=Mtanh(mL_C) if there is convective heat transfer at the tip

where $m=\sqrt{\frac{hP}{kA_c}}$, and $M=\sqrt{hPkA_C}\,q_b = \sqrt{hPkA_C}\,(T_b - T_\infty)$

Use the thermal resistance concept:

$$q_f = \sqrt{hPkA_C}\,\tanh(mL)(T_b - T_\infty) = \frac{(T_b - T_\infty)}{R_{t,f}}$$

where $R_{t,f}$ is the thermal resistance of the fin.

For a fin with an adiabatic tip, the fin resistance can be expressed as

$$R_{t,f} = \frac{(T_b - T_\infty)}{q_f} = \frac{1}{\sqrt{hPkA_C}\,[\tanh(mL)]}$$

Fin Effectiveness

How effective a fin can enhance heat transfer is characterized by the fin effectiveness ε_f: Ratio of fin heat transfer and the heat transfer without the fin. For an adiabatic fin:

$$e_f = \frac{q_f}{q} = \frac{q_f}{hA_C(T_b - T_\infty)} = \frac{\sqrt{hPkA_C}\tanh(mL)}{hA_C} = \sqrt{\frac{kP}{hA_C}}\tanh(mL)$$

If the fin is long enough, mL>2, $\tanh(mL) \rightarrow 1$,

it can be considered an infinite fi)

$$e_f \rightarrow \sqrt{\frac{kP}{hA_C}} = \sqrt{\frac{k}{h}\left(\frac{P}{A_C}\right)}$$

In order to enhance heat transfer, $e_f > 1$.

However, $e_f \geq 2$ will be considered justifiable

If $e_f < 1$ then we have an insulator instead of a heat fin

Fin Effectiveness (cont.)

$$e_f \rightarrow \sqrt{\frac{kP}{hA_C}} = \sqrt{\frac{k}{h}\left(\frac{P}{A_C}\right)}$$

• To increase ε_f, the fin's material should have higher thermal conductivity, k.

• It seems to be counterintuitive that the lower convection coefficient, h, the higher ε_f. But it is not because if h is very high, it is not necessary to enhance heat transfer by adding heat fins. Therefore, heat fins are more effective if h is low. Observation: If fins are to be used on surfaces separating gas and liquid. Fins are usually placed on the gas side. (Why?)

• P/A_C should be as high as possible. Use a square fin with a dimension of W by W as an example: $P=4W$, $A_C=W^2$, $P/A_C=(4/W)$. The smaller W, the higher the P/A_C, and the higher $\varepsilon_{f.}$

•Conclusion: It is preferred to use thin and closely spaced (to increase the total number) fins.

Fin Effectiveness (cont.)

The effectiveness of a fin can also be characterized as

$$\varepsilon_f = \frac{q_f}{q} = \frac{q_f}{hA_C(T_b - T_\infty)} = \frac{(T_b - T_\infty)/R_{t,f}}{(T_b - T_\infty)/R_{t,h}} = \frac{R_{t,h}}{R_{t,f}}$$

It is a ratio of the thermal resistance due to convection to the thermal resistance of a fin. In order to enhance heat transfer, the fin's resistance should be lower than that of the resistance due only to convection.

Fin Efficiency

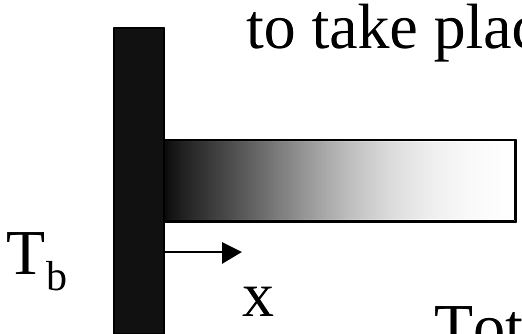
Define Fin efficiency: $h_{\mathrm{f}} = \dfrac{q_f}{q_{\max}}$

where $q_{\max}$ represents an idealized situation such that the fin is made up of material with infinite thermal conductivity. Therefore, the fin should be at the same temperature as the temperature of the base.

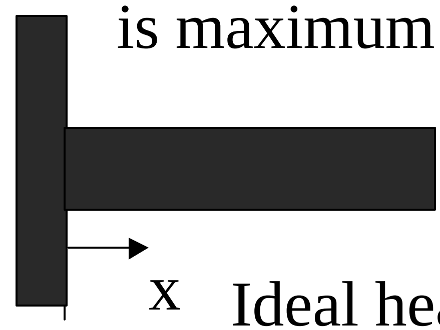
$$q_{\max} = hA_f(T_b - T_\infty)$$

$T(x)<T_b$ for heat transfer to take place

For infinite k $T(x)=T_b$, the heat transfer is maximum



Total fin heat transfer q_f



Ideal heat transfer q_{max}

Real situation

Ideal situation

Fin Efficiency (cont.)

Use an adiabatic rectangular fin as an example:

$$\eta_f = \frac{q_f}{q_{max}} = \frac{M \tanh mL}{hA_f(T_b - T_\infty)} = \frac{\sqrt{hPkA_c}(T_b - T_\infty)\tanh mL}{hPL(T_b - T_\infty)}$$

$$= \frac{\tanh mL}{\sqrt{\frac{hP}{kA_c}}L} = \frac{\tanh mL}{mL}$$ (see ~~Table 3.5~~ for η_f of common fins)

Figures 8-59, 8-60

The fin heat transfer: $q_f = \eta_f q_{max} = \eta_f hA_f(T_b - T_\infty)$

$$q_f = \frac{T_b - T_\infty}{1/(\eta_f hA_f)} = \frac{T_b - T_\infty}{R_{t,f}}, \text{ where } R_{t,f} = \frac{1}{\eta_f hA_f}$$

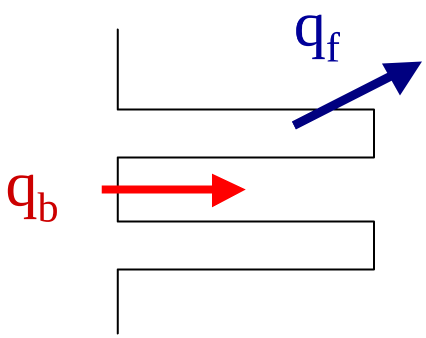
Thermal resistance for a single fin.

As compared to convective heat transfer: $R_{t,b} = \frac{1}{hA_b}$

In order to have a lower resistance as that is required to enhance heat transfer: $R_{t,b} > R_{t,f}$ or $A_b < \eta_f A_f$

Overall Fin Efficiency

Overall fin efficiency for an array of fins:



Define terms: A_b: base area exposed to coolant

A_f: surface area of a single fin

A_t: total area including base area and total finned surface, $A_t = A_b + NA_f$

N: total number of fins

$$q_t = q_b + Nq_f = hA_b(T_b - T_\infty) + N\eta_f hA_f(T_b - T_\infty)$$

$$= h[(A_t - NA_f) + N\eta_f A_f](T_b - T_\infty) = h[A_t - NA_f(1 - \eta_f)](T_b - T_\infty)$$

$$= hA_t[1 - \frac{NA_f}{A_t}(1 - \eta_f)](T_b - T_\infty) = \eta_O hA_t(T_b - T_\infty)$$

Define overall fin efficiency: $\eta_O = 1 - \frac{NA_f}{A_t}(1 - \eta_f)$

Heat Transfer from a Fin Array

$$q_t = hA_t h_O (T_b - T_\infty) = \frac{T_b - T_\infty}{R_{t,O}} \text{ where } R_{t,O} = \frac{1}{hA_t h_O}$$

Compare to heat transfer without fins

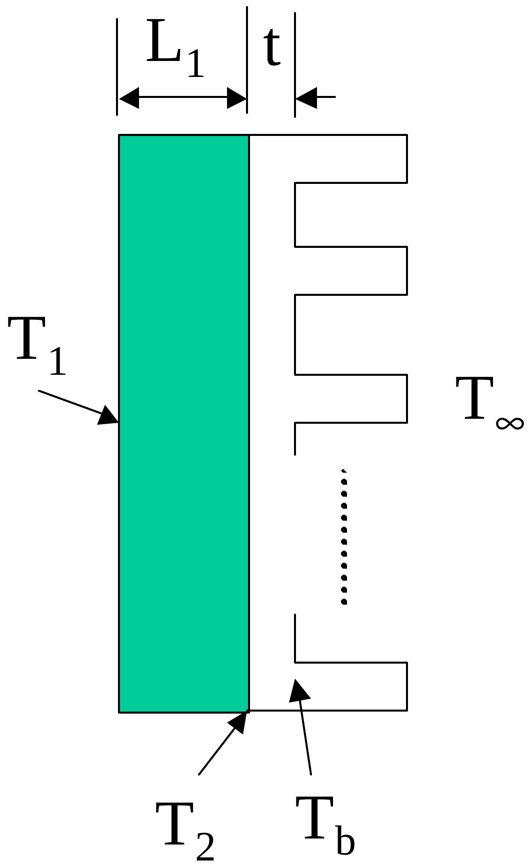
$$q = hA(T_b - T_\infty) = h(A_b + NA_{b,f})(T_b - T_\infty) = \frac{1}{hA}$$

where $A_{b,f}$ is the base area (unexposed) for the fin

To enhance heat transfer $A_t h_O >> A = A_b + NA_{b,f}$

That is, to increase the effective area $h_O A_t$.

Thermal Resistance Concept



$A = A_b + NA_{b,f}$

$R_b = t/(k_b A)$

$R_1 = L_1/(k_1 A)$

$R_{t,O} = 1/(hA_t h_O)$

$$q = \frac{T_1 - T_\infty}{\sum R} = \frac{T_1 - T_\infty}{R_1 + R_b + R_{t,O}}$$