Basic Fluid Properties and Governing Equations

- Density (ρ): mass per unit volume (kg/m^3 or $slug/ft^3$)
- Specific Volume ($v=1/\rho$): volume per unit mass
- Temperature (T): thermodynamic property that measures the molecular activity of an object. It is used to determine whether an object has reached thermal equilibrium.
- Pressure (p):pressure can be considered as an averaged normal force exerted on a unit surface area by impacting molecules.

($P = \lim_{A \to 0}\left(\frac{F}{A}\right)$; N/m^2 or pascal; lb/in^2 or psi)

Pascal law: (under static condition) pressure acts uniformly in all directions. It also acts perpendicular to the containing surface.

If a fluid system is not in motion, then the fluid pressure is equal its thermodynamic pressure.

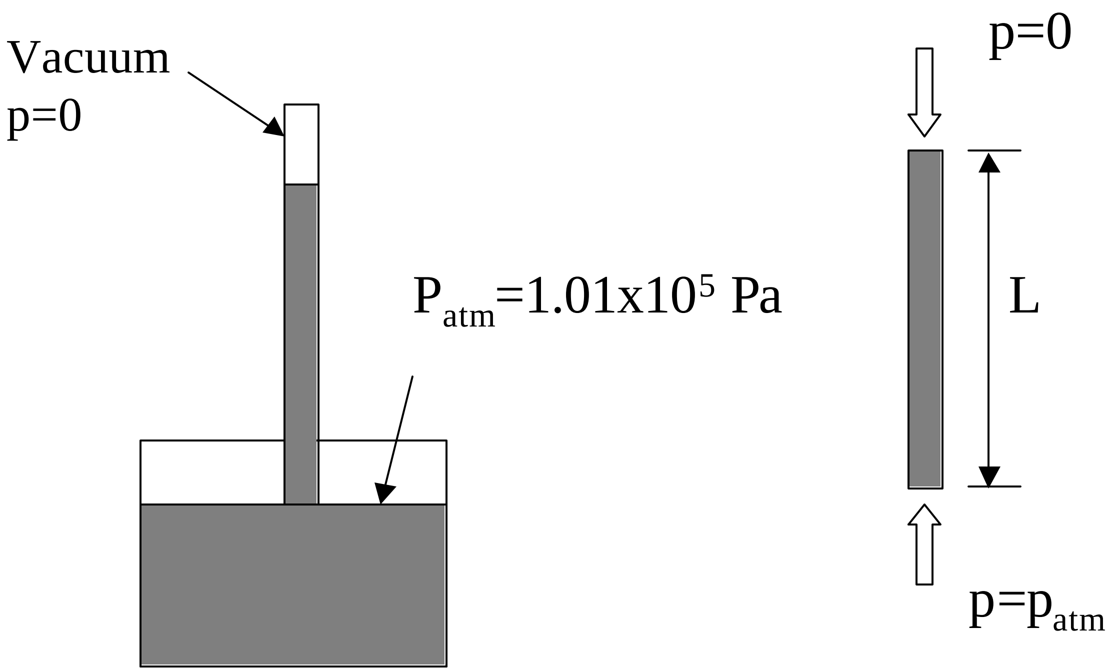
- Atomspheric pressure (p_{atm}): pressure measured at the earth's surface.

1 atm = 14.696 psi = 1.01325 x 105 N/m^2 (pascal)

- Absolute pressure: pressure measured without reference to other pressures.

Gage pressure: $p_{gage} = p_{absolute} - p_{atm}$

Atmospheric pressure can be measured using a barometer:



Force balance

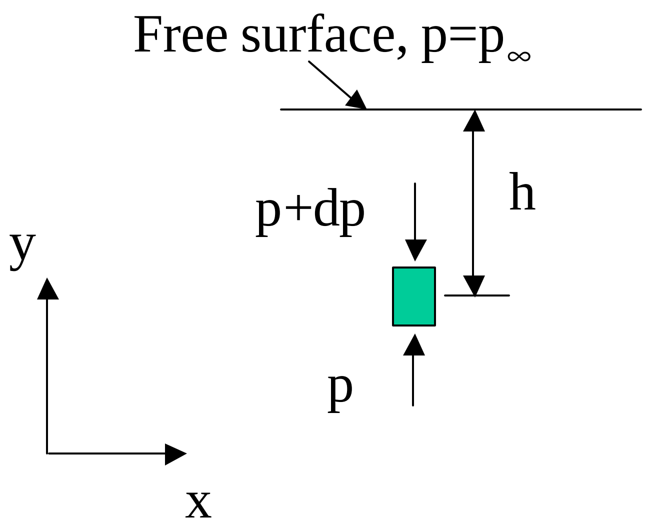
$$p_{atm}A = W = mg = rALg$$

$$P_{atm} = rgL$$

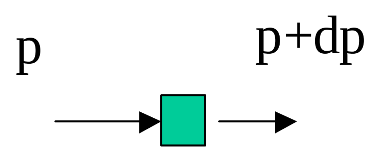
r is the density of the fluid, g is the gravitatio nal constant

Similarly, this balance can be applied to a small fluid element as shown

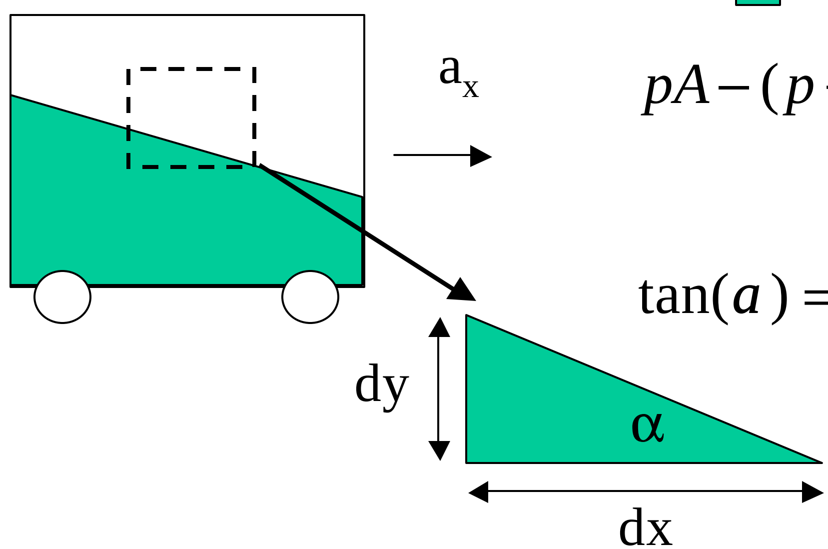
$$pA-(p+dp)A=mg=rAgdy,\ \frac{dp}{dy}=-rg,$$ integrate from fluid element to the free surface $p(h)=p_\infty+rgh$



Example: If a container of fluid is accelerating with an acceleration of a_x to the right as shown below, what is the shape of the free surface of the fluid?

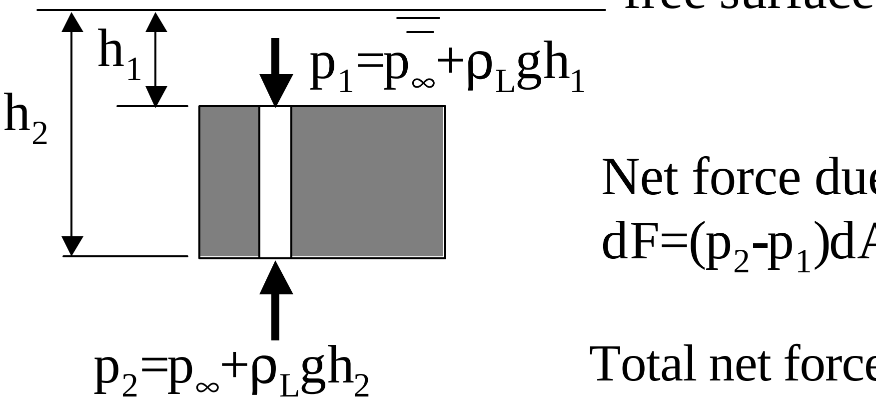


$$pA-(p+dp)A=ma_x=rAdxa_x,-\frac{dp}{dx}=ra_x$$



$$\tan(a)=\frac{dy}{dx}=\frac{dp/rg}{dp/ra_x}=\frac{a_x}{g},\ a=\tan^{-1}\left(\frac{a_x}{g}\right)$$

Buoyancy of a submerged body



Net force due to pressure difference

$dF=(p_2-p_1)dA=\rho_L g(h_2-h_1)dA$

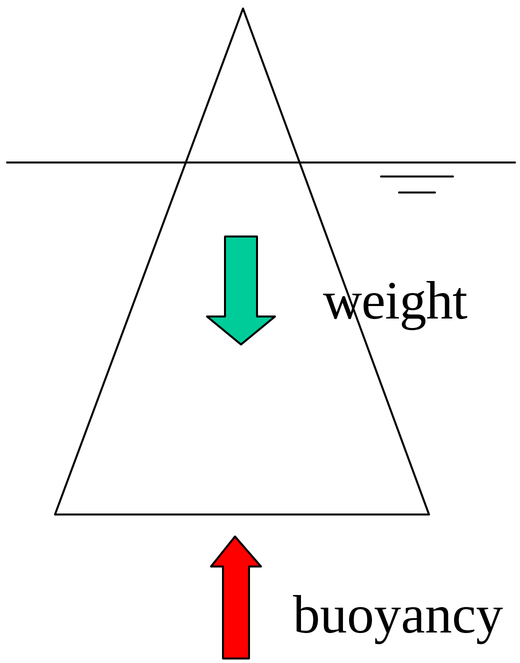
Total net force (buoyancy)

$$F_B = \int dF = \rho_L g \int (h_2 - h_1) dA = \rho_L g \forall_{displaced}$$

The principle of Archimedes:
The buoyancy acting on a submerged object is equal to the weight of the displaced fluid due to the presence of the object.

This law is valid for all fluid and regardless of the shape of the body. It can also be applied to both fully and partially submerged bodies.

Example: Titanic sank when it struck an iceberg on April 14, 1912. Five of its 16 watertight compartments were punctuated when it collides with the iceberg underwater. Can you estimate the percentage of the iceberg that is actually beneath the water surface? It is known that when water freezes at 0° C, it expands and its specific gravity changes from 1 to 0.917.



When the iceberg floats, its weight balances the buoyancy force exerted on the iceberg by the displaced water.

$$W = F_B$$

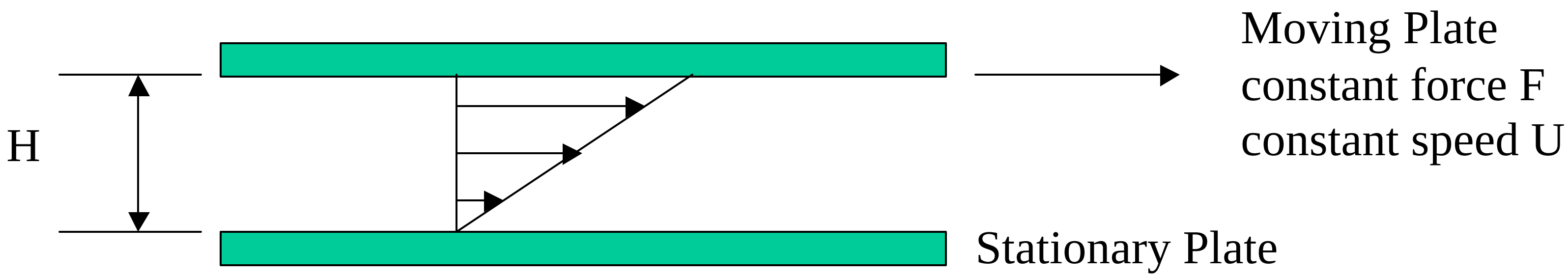
$$r_{ice}\, g\, \cancel{V}_{ice-berg} = r_{water}\, g\, \cancel{V}_{submerged}$$

$$\frac{\cancel{V}_{submerged}}{\cancel{V}_{ice-berg}} = \frac{r_{ice}}{r_{water}} = \frac{0.917}{1} = 91.7\%$$

Therefore, more than 90% of the iceberg is below the water surface.

Properties (cont.)

- Viscosity: Due to interaction between fluid molecules, the fluid flow will resist a shearing motion. The viscosity is a measure of this resistance.



From experimental observation, F ∝ A(U/H)=A(dV/dy)

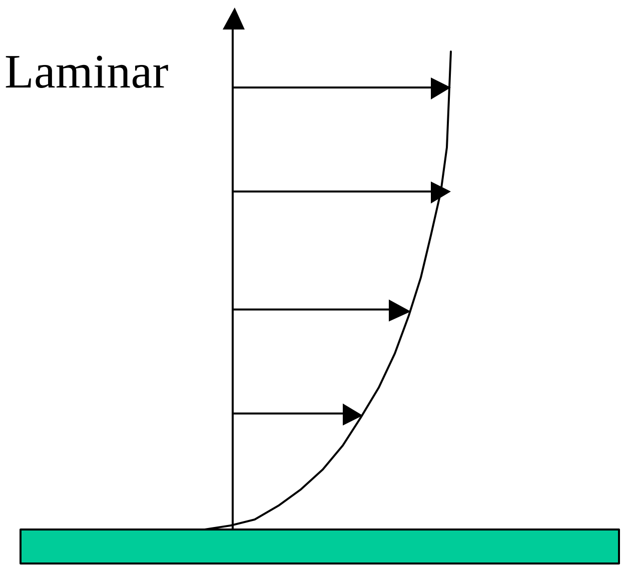
$$t = \frac{F}{A} \propto \frac{dV}{dy}, \text{ where } t \text{ is shear stress}$$

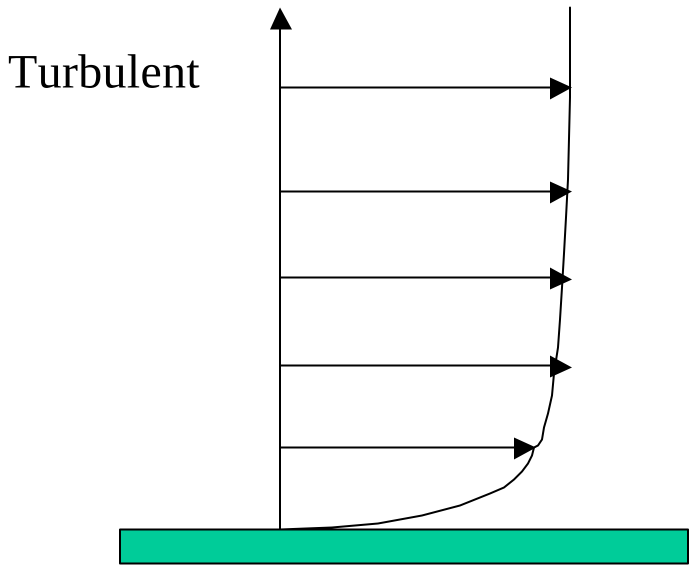
$$t = m\frac{\mathrm{dV}}{\mathrm{dy}}, \text{ where } m \text{ is dynamic viscosity, The unit of } m \text{ is } \frac{lb\,\mathrm{sec}}{ft^2} \text{ or } \frac{N\,\mathrm{sec}}{m^2}$$

$$\text{kinematic viscosity } n = \frac{m}{r}, \text{ has unit of } \frac{\mathrm{ft}^2}{\mathrm{sec}}\; or \;\frac{m^2}{\mathrm{sec}}$$

Boundary Layer Concept

Immediately adjacent to a solid surface, the fluid particles are slowed by the strong shear force between the fluid particles and the surface. This relatively slower moving layer of fluid is called a “boundary layer”.





$$t = m\frac{dV}{dy}$$

Question: which profile has larger wall shear stress?
In other words, which profile produces more frictional drag against the motion of the solid surface?

Partial Differential Equations (PDE): Many physical phenomena are governed by PDE since the physical functions involved usually depend on two or more independent variables (ex. Time, spatial coordinates). Their variation with respect to these variables need to be described by PDE not ODE (Ordinary Differential Equations).

Example: In dynamics, we often track the change of the position of an object in time. Time is the only variable in this case. X=x(t), u=dx/dt, a=du/dt.

In heat transfer, temperature inside an object can vary with both time and space. T=T(x,t). The temperature varies with time since it has not reach its thermal equilibrium.

$$rC_p \frac{\partial T}{\partial t} = q_{in} - q_{out} \neq 0$$

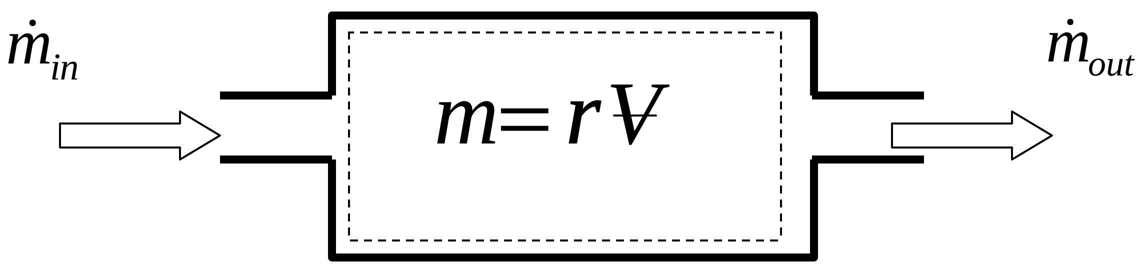
The temperature can also vary in space as according to the Fourier's law:

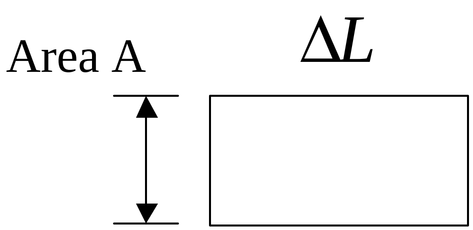
$$q = -KA\frac{\partial T}{\partial x}, \text{ if q} \neq 0, \text{ then } \frac{\partial \mathrm{T}}{\partial \mathrm{x}} \neq 0$$

Basic equations of Fluid Mechanics

- Mass conservation (continuity equation):
  - The rate of mass stored = the rate of mass in - the rate of mass out

$$\frac{dm}{dt} = \dot{m}_{in} - \dot{m}_{out}$$





Within a given time Δt, the fluid element with a cross-sectional area of A moves a distance of ΔL as shown.
The mass flow rate can be represented as

$$\dot{m} = \frac{\Delta m}{\Delta t} = rA\frac{\Delta L}{\Delta t} = rAV$$

$$\frac{dm}{dt} = \frac{d(r\cancel{V})}{dt} = \boxed{r\frac{d\cancel{V}}{dt}} + \boxed{\cancel{V}\frac{dr}{dt}} = \dot{m}_{in} - \dot{m}_{out}$$

Keep density constant,
volume changes with time
ex: blow up a soap bubble

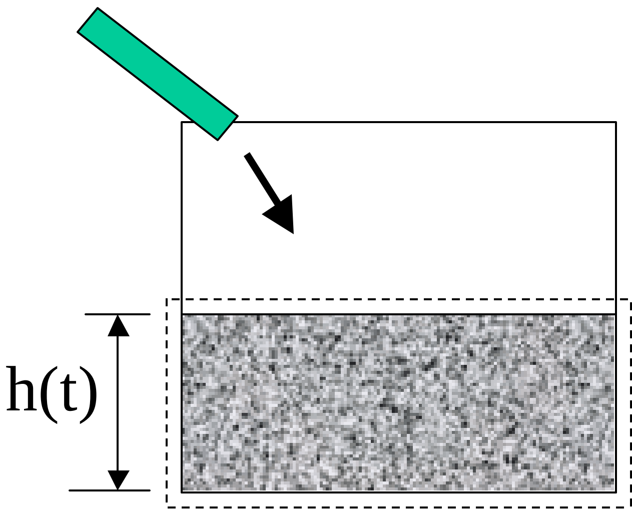
Keep volume constant,
density changes with time
ex: pump up a basketball

$$\frac{dm}{dt} = (rAV)_{in} - (rAV)_{out}$$

For steady state condition: mass flow in = mass flow out

$$(rAV)_{in} = (rAV)_{out}$$

Examples: filling up an empty tank

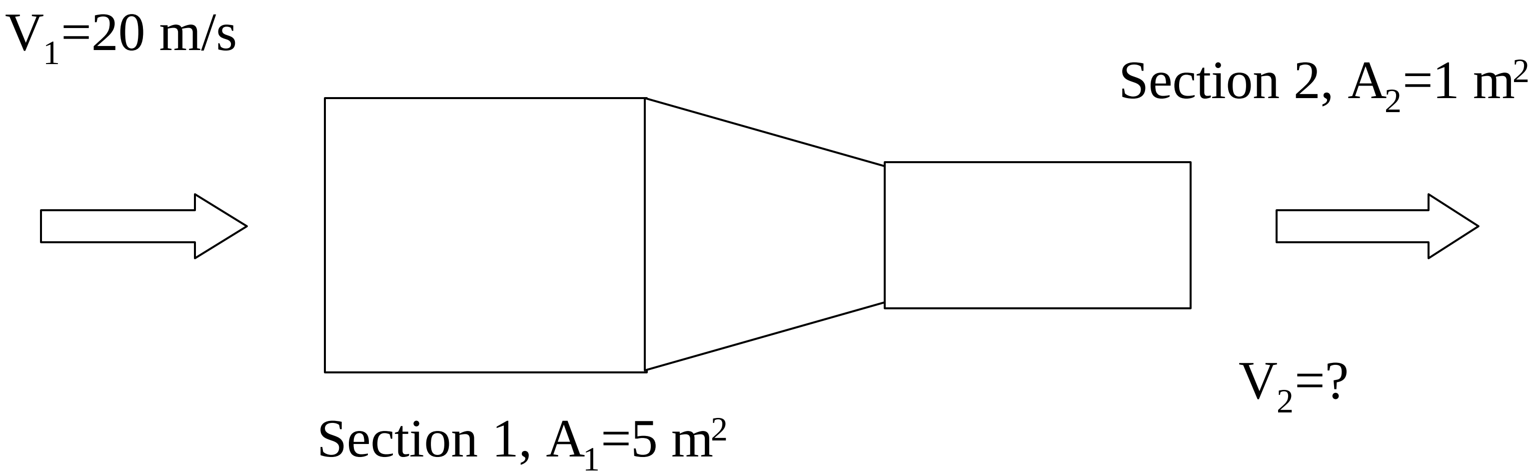


Water is fed into an empty tank using a hose of cross-sectional area of 0.0005 m^2. The flow speed out of the hose is measured to be 10 m/s. Determine the rate of increase for the water height inside the tank dh/dt. The cross-sectional area of the tank is 1 m^2.

0, no mass out

$$\frac{dm}{dt} = r\frac{dV}{dt} = rA_{\tan k}\frac{dh}{dt} = \dot{m}_{in} - \boxed{\dot{m}_{out}} = rA_{hose}V$$

$$\frac{dh}{dt} = \left(\frac{A_{hose}}{A_{\tan k}}\right)V = \left(\frac{0.0005}{1}\right)(10) = 0.005(m/s)$$



$$\dot{m}_{in} = \dot{m}_{out}$$

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2, \text{ for constant density}$$

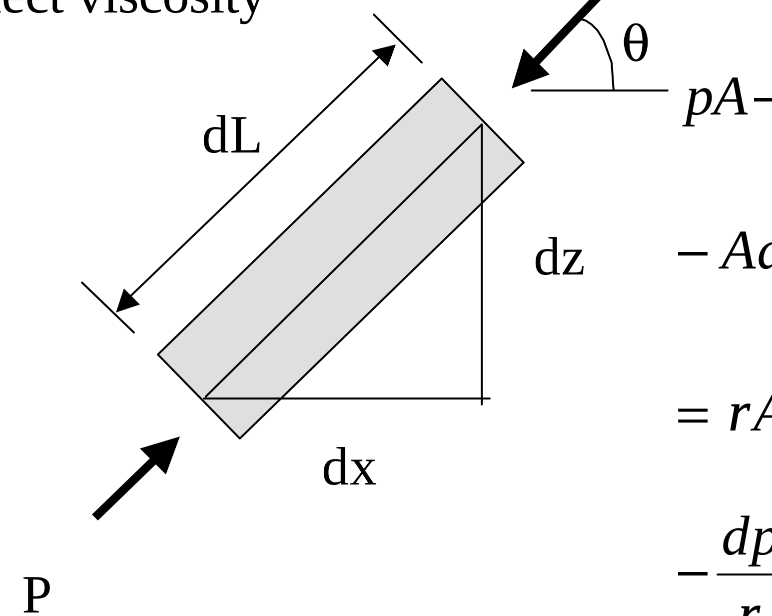
$$V_2 = \left(\frac{A_1}{A_2}\right) V_1 = \left(\frac{5}{1}\right)(20) = 100(m/s)$$

•Momentum Conservation: (Newton’s second law)

Net external forces lead to the change of linear momentum

$$\sum \vec{F} = \frac{d}{dt}(m\vec{V})$$

First, neglect viscosity



$$pA - (p + dp)A - W\sin q = \frac{d}{dt}(mV)$$

$$-Adp - rA(dL)g\left(\frac{dz}{dL}\right) = rA(dL)\left(\frac{dV}{dt}\right)$$

$$= rA(dL)\left(\frac{dV}{dL}\frac{dL}{dt}\right) = rA(VdV)$$

$$-\frac{dp}{r} - gdz = VdV$$

$$\boxed{\frac{dp}{r} + VdV + gdz = 0}$$

Euler’s Equation

Euler's Equation: $\boxed{\dfrac{dp}{r} + VdV + gdz = 0}$

- First, dz=0 no elevation variation $\boxed{\dfrac{dp}{r} + VdV = 0}$

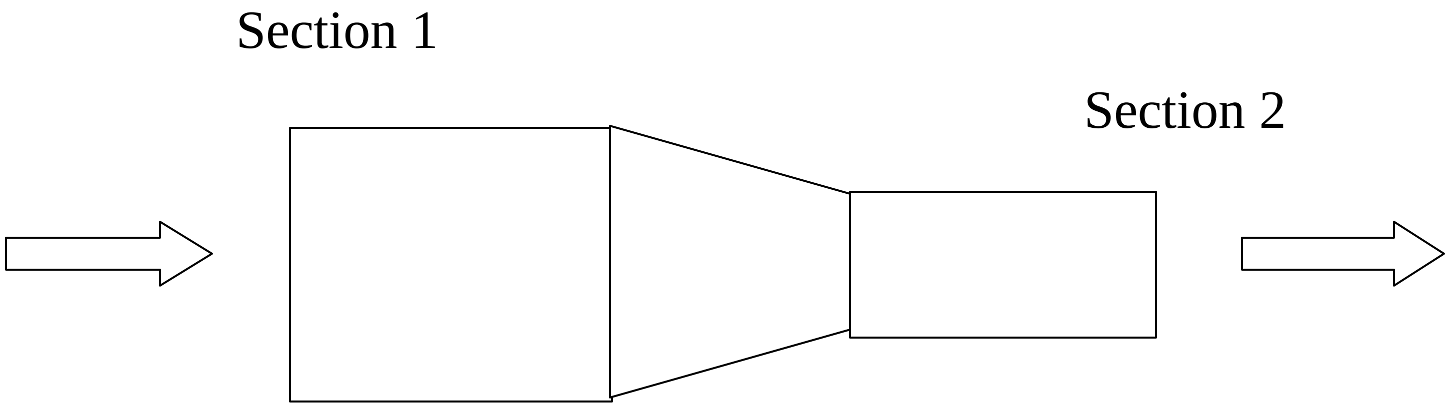
If dp>0, pressure increases as fluid flows downstream
then dV<0, velocity decreases due to the adverse pressure gradient and vice versa.

- If dp=0, no external pressure gradient $\boxed{VdV + gdz = 0}$

If dz<0, fluid flows to a lower point, dV>0, its velocity increases and vice versa

- If dV=0, no flow $\boxed{\dfrac{dp}{r} + gdz = 0}$

If dz<0, into the lower elevation inside the static fluid system, dp>0, pressure increases



Air flows through a converging duct as shown. The areas at sections 1 & 2 are 5 m^3 and 1 m^3, respectively. The inlet flow speed is 20 m/s and we know the outlet speed at section 2 is 100 m/s by mass conservation. If the pressure at section 2 is the atmospheric pressure at 1.01x10^5 N/m^2, what is the pressure at section 1. Neglect all viscous effects and given the density of the air as 1.185 kg/m^3.

$$\frac{dp}{r} + VdV = 0, \text{integrate from section 1 to section 2}$$

$$\int_1^2 \frac{dp}{r} = \int_1^2 -VdV, \Rightarrow \frac{p_2 - p_1}{r} = \frac{V_1^2 - V_2^2}{2}$$

$$p_1 = p_2 + \left(\frac{r}{2}\right)\left(V_2^2 - V_1^2\right)$$

$$= 1.01\times10^5 + \left(\frac{1.185}{2}\right)\left(100^2 - 20^2\right) = 1.01\times10^5 + 5688(Pa)$$