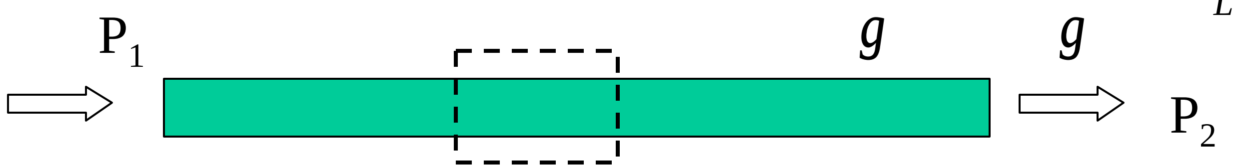
Frictional losses in piping system

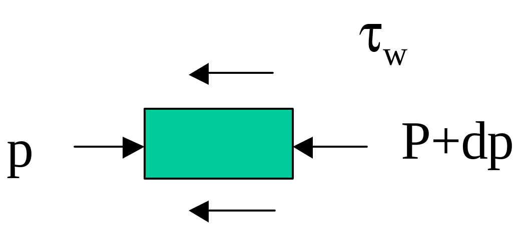
Extended Bernoulli's equation, $\frac{p_1}{g}+\frac{V_1^2}{2g}+z_1+h_A-h_E-h_L=\frac{p_2}{g}+\frac{V_2^2}{2g}+z_2$

$$\frac{p_1-p_2}{g}=\frac{\Delta p}{g}=h_L = \text{frictional head loss}$$



Consider a laminar, fully developed circular pipe flow

R: radius, D: diameter
L: pipe length
τ_w: wall shear stress



$$[p-(p+dp)](p R^2)=t_w(2p R)dx,$$

Pressure force balances frictional force

$$-dp=\frac{2t_w}{R}dx, \text{ integrate from 1 to 2}$$

Darcy's Equation: $\frac{\Delta p}{g}=\frac{p_1-p_2}{g}=h_L=\frac{4t_w}{r g}\left(\frac{L}{D}\right)=f\left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right)$

$$t_w=\left(\frac{f}{4}\right)\left(\frac{rV^2}{2}\right)$$

where f is defined as frictional factor characterizing pressure loss due to pipe wall shear stress

When the pipe flow is laminar, it can be shown (not here) that

$f = \dfrac{64m}{VDr}$, by recognizing that $\mathrm{Re} = \dfrac{rVD}{m}$,as Reynolds number

Therefore, $f = \dfrac{64}{\mathrm{Re}}$, frictional factor is a function of the Reynolds number

Similarly, for a turbulent flow, f=function of Reynolds number also

$f = F(\mathrm{Re})$. Another parameter that influences the friction is the surface roughness as relativeto the pipe diameter $\dfrac{e}{\mathrm{D}}$.

Such that $f = F\left(\mathrm{Re}, \dfrac{e}{\mathrm{D}}\right)$: Pipe frictional factor is a function of pipe Reynolds number and the relative roughness of pipe.

This relation is sketched in the Moody diagram as shown in the following page.

The diagram shows f as a function of the Reynolds number (Re), with a series of parametric curves related to the relative roughness $\left(\dfrac{e}{\mathrm{D}}\right)$.