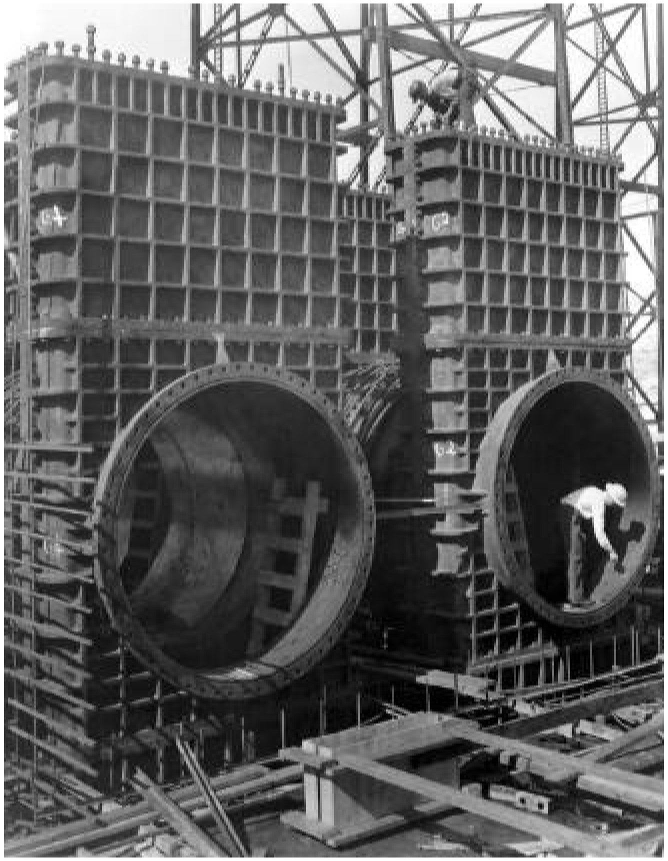
Application Examples

Feeder Gates for Canal

Gate Valves for Spillway Control

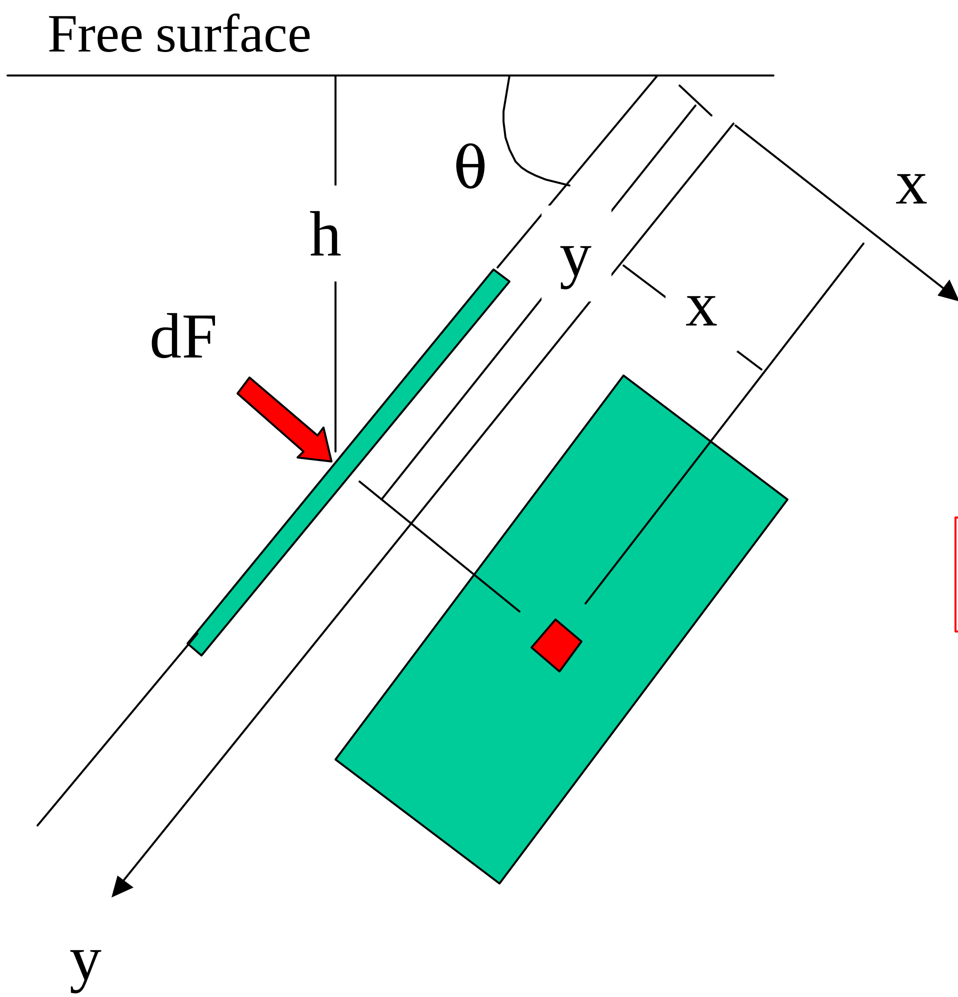


Applications (cont.)

Spillway Drum Gates: hollow inside, use buoyancy to control the position of the gate.

Hydrostatic Force on an Inclined Plane Surface

Assume atmoshperic condition on the other side of the surface



$dF = PdA = g\,hdA = r\,ghdA$

$= r\,gy\sin q\,dA$

Integrate over the entire surface

$F_R = \int dF = r\,g\sin q \int ydA$

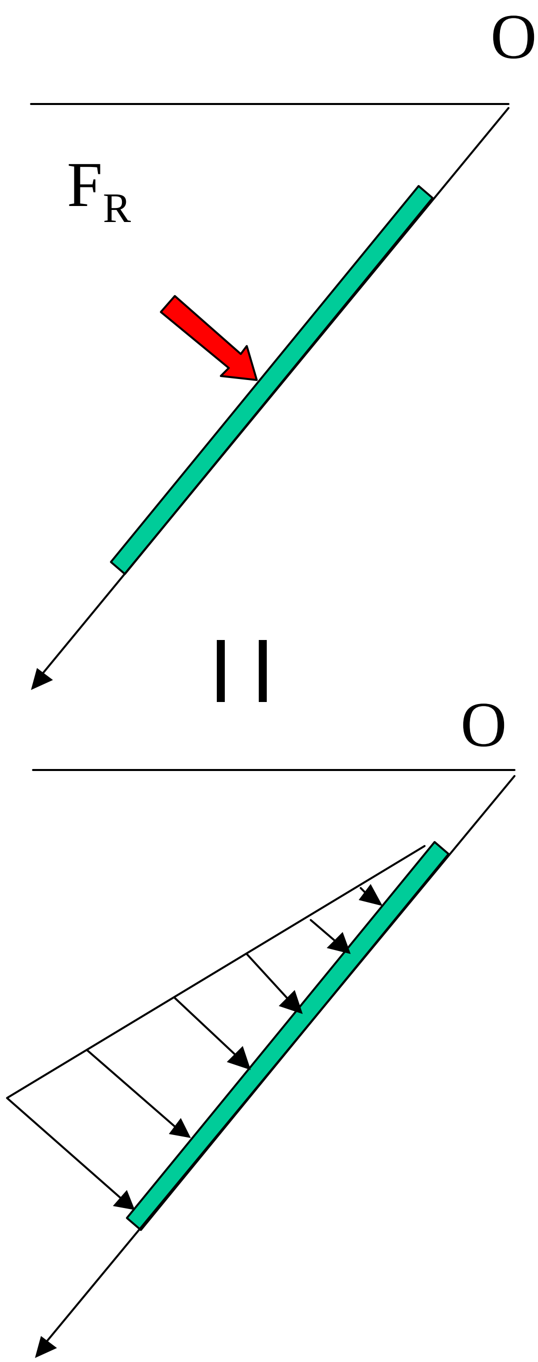
Define centroid of the area y_C

$y_C = \dfrac{1}{A}\int ydA$, so that

$$\boxed{F_R = r\,gAy_C \sin q = r\,gAh_C}$$

In order to find equilavent system, need to make sure that the moment of the resultant force must equal to the moment of the distributed force.

Hydrostaic forces



Taking Mmoment about the x-axis: $y'F_R = \int_A yP\,dA$

$$y'\rho g \sin\theta\, y_C A = \int_A \rho g y^2 \sin\theta\, dA = \rho g \sin\theta \int_A y^2 dA$$

Recognize that $\int_A y^2 dA = I_{xx}$ (area moment of inertia about O)

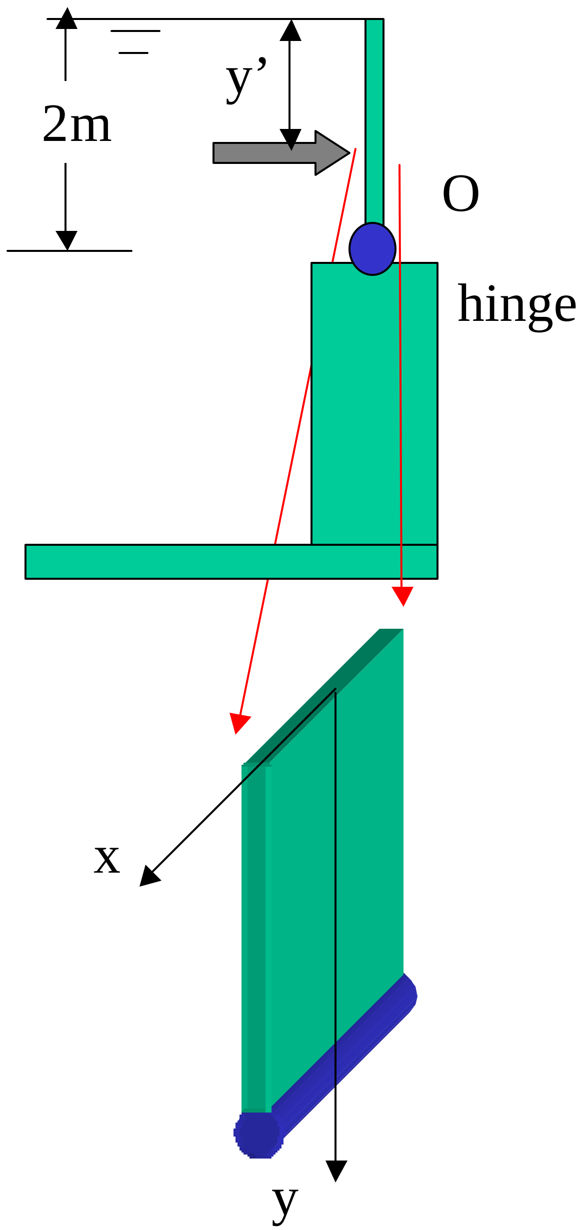
Therefore, $y' = \dfrac{I_{xx}}{Ay_C}$

Also, from parallel axis theorem, we can relate I_{xx} to $I_{\hat{x}\hat{x}}$, moment of inertia about the centroid of the area (can be found in table)

$I_{xx} = I_{\hat{x}\hat{x}} + Ay_C^2$, therefore, $y' = y_C + \dfrac{I_{\hat{x}\hat{x}}}{Ay_C}$

Similarly, $x' = \dfrac{I_{xy}}{Ay_C} = x_C + \dfrac{I_{\hat{x}\hat{y}}}{Ay_C}$

Example



The square flood gate (2m by 2m) is hinged along its bottom as shown. Determine the moment at the hinge in order to hold the gate steady.

First, find the resultant force:

$$F_R = \rho g h_C A = (1000)(9.8)(1)(2\times 2) = 39200(N)$$

Then, determine the point of action:

$$y' = y_C + \frac{I_{\hat{x}\hat{x}}}{A y_C} = (1) + \frac{\frac{1}{12}(2)(2)^3}{(2\times 2)(1)} = 1 + \frac{1}{3} = \frac{4}{3}(m)$$

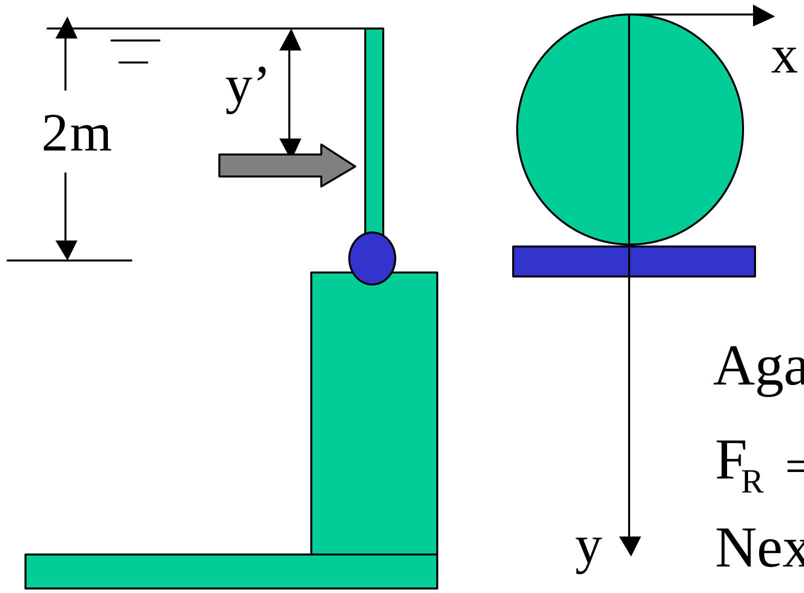
As expected, it falls at a depth 2/3 of the total depth.

The holding moment (M) on the hinge O will be

$$\sum M_O = M - F_R\left(2 - \frac{4}{3}\right) = 0,$$

$$M = 18479(N.m)$$

Example (cont.)



If the square gate is replaced by a circular-shaped gate as shown, recalculate the holding moment.

Again, find the resultant force first:

$$F_R = \rho g h_C A = (1000)(9.8)(1)\pi(1)^2 = 30772(N)$$

Next, the line of action:

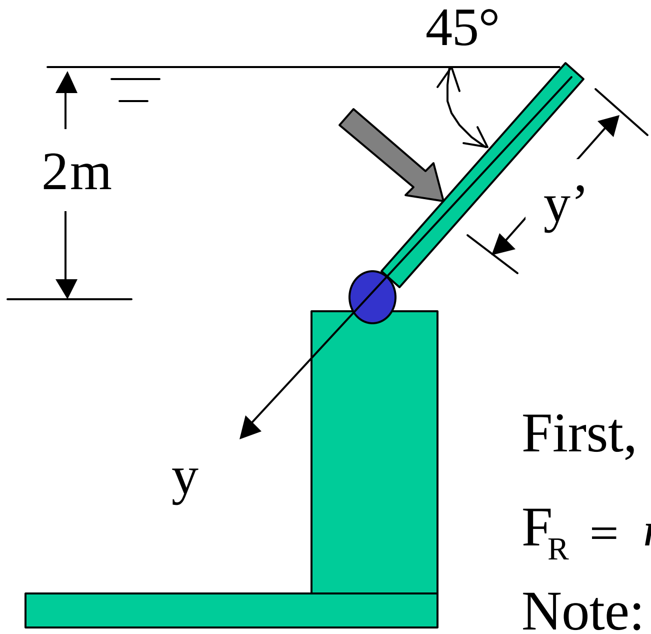
$$y' = y_C + \frac{I_{\hat{x}\hat{x}}}{A y_C} = 1 + \frac{\frac{1}{4}\pi R^4}{\pi R^2(1)} = 1 + \frac{1}{4} = \frac{5}{4}(m)$$

The holding moment:

$$\sum M_O = M - F_R\left(2 - \frac{5}{4}\right) = M - \frac{3}{4}F_R = 0$$

$$M = 23079(N.m)$$

Example (cont.)



If the square gate is placed at an angle of 45° as shown, recalculate the holding moment again. Note: the y axis has been redefined to follow the gate for convenience.

First, calculate the resultant force:

$$F_R = \rho g h_C A = (1000)(9.8)(1)(2\sqrt{2}\times 2\sqrt{2}) = 78400(N)$$

Note: the h stays the same and is independent of the incline angle, however, the gate area increases.

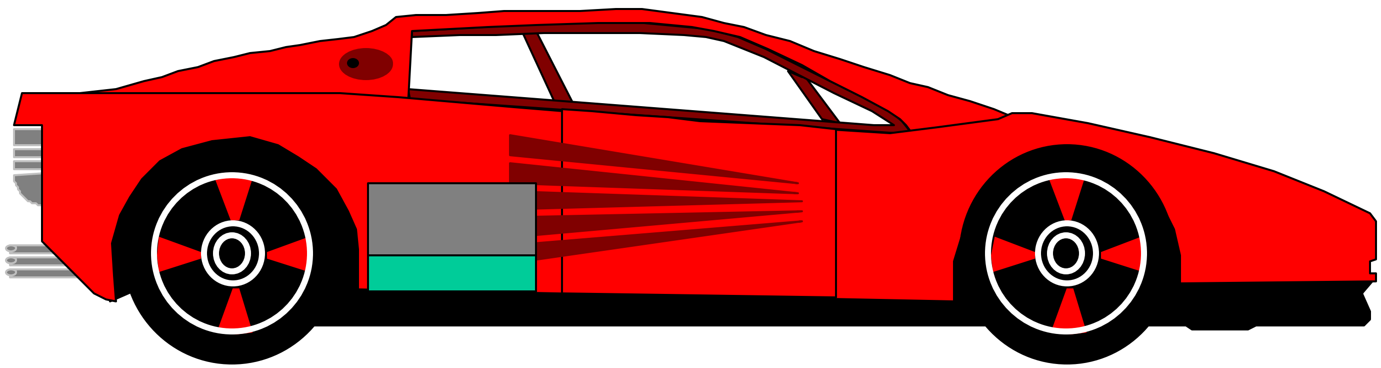
$$y' = y_C + \frac{I_{\hat{x}\hat{x}}}{A y_C} = \frac{2\sqrt{2}}{2} + \frac{\frac{1}{12}(2\sqrt{2})(2\sqrt{2})^3}{(2\sqrt{2}\times 2\sqrt{2})\left(\frac{2\sqrt{2}}{2}\right)}$$

$$y' = \sqrt{2} + \frac{\sqrt{2}}{3} = \frac{4\sqrt{2}}{3}$$

The holding moment: $M = F_R\,(2\sqrt{2} - \frac{4\sqrt{2}}{3}) = 73916(N.m)$

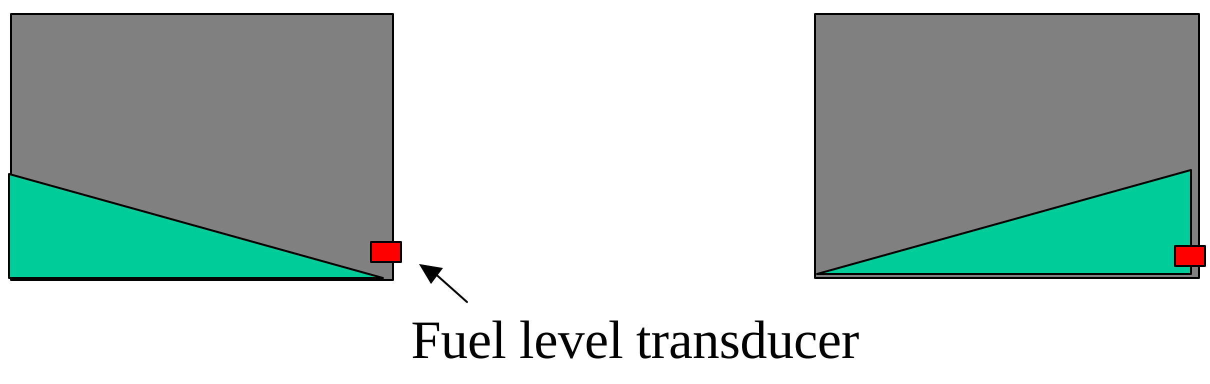
An interesting observation

When the gas tank is low, the low fuel light will lit to warn the driver. Have you noticed that the light will not always stay on for a period of time. It turns off when either you accelerate (decelerate) or climb (descend) on a sloped road. Can you explain this phenomenon by using the principle of fluid statics.



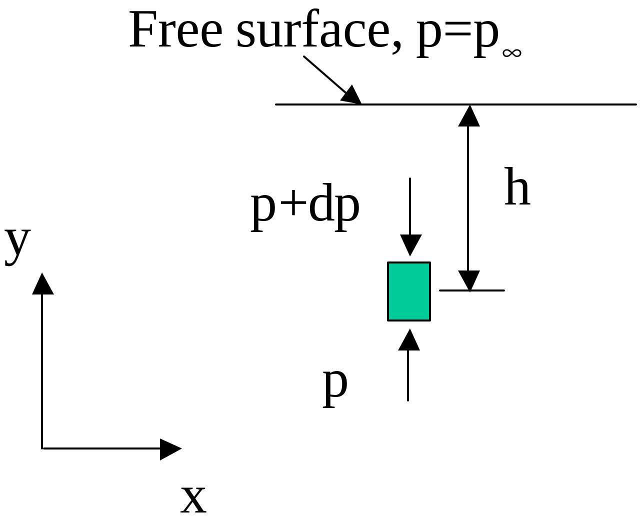
Accelerating (climbing)

Decelerating (descending)



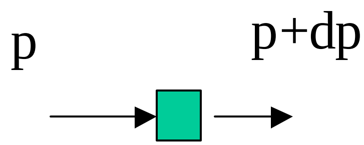
Hydrostatic balance can be applied to a small fluid element as shown

$pA - (p + dp)A = mg = r\,Agdy,\ \dfrac{dp}{dy} = -r\,g$,integrate from fluid element to

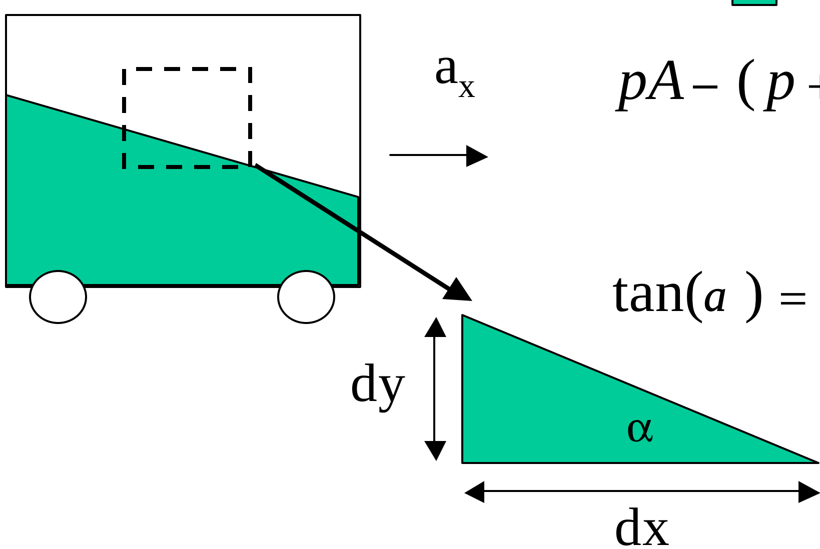


the free surface $p(h) = p_\infty + r\,gh$

Example: If a container of fluid is accelerating with an acceleration of a_x to the right as shown below, the free surface of the fluid will incline with an angle α as shown.



$$pA - (p + dp)A = ma_x = r\,Adxa_x,\ -\frac{dp}{dx} = r\,a_x$$



$$\tan(a) = \frac{dy}{dx} = \frac{dp/r\,g}{dp/r\,a_x} = \frac{a_x}{g},\ a = \tan^{-1}\left(\frac{a_x}{g}\right)$$