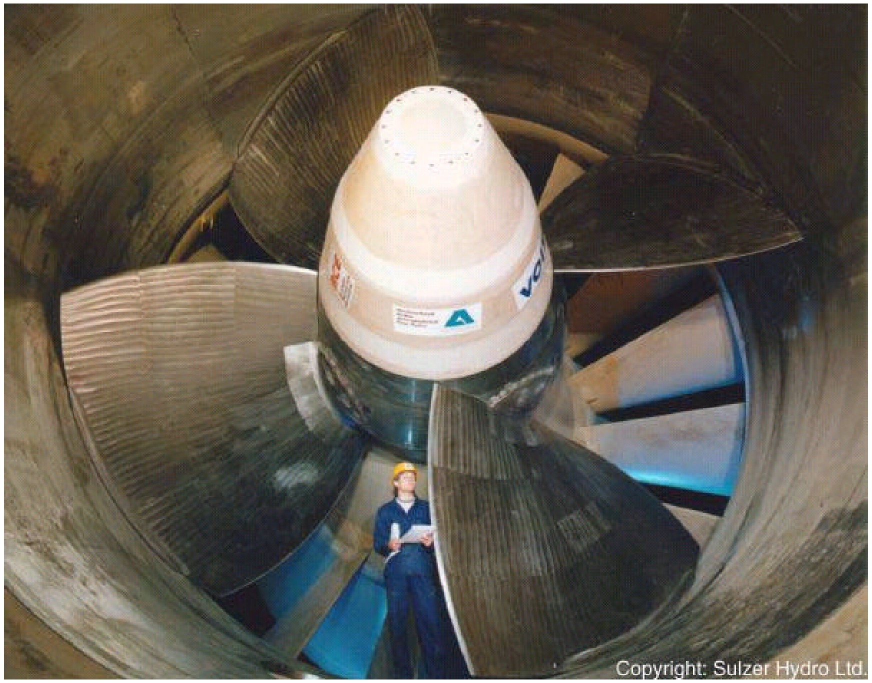
Practical Applications

Wind Turbine

Hydropower Turbine



The motion of a fluid is altered so that propulsive forces can be generated on the devices. The integral linear momentum equation is used to determine the forces acting on such devices. Other examples: lift and drag forces on wings, pressure surge across a compressor.

Conservation Equations

Similar derivation can be applied to obtain other conservation equations such as conservation principles of linear momentum, energy and angular momentum.

First, the linear momentum equation is based on the Newton's second law: The sum of all external forces actin on a system is equal to the time rate change of the linear momentum of the system.

$\vec{F} = (\frac{d\vec{P}}{dt})_{system}$, where the linear momentum of the system is given by

$$\vec{P} = \int_{system} r\vec{V} d\forall.$$

Using Reynolds transport theorem, it is obtained that

$$(\frac{d\vec{P}}{dt})_{system} = [\frac{\partial}{\partial t}\int_{CV} r\vec{V} d\forall] + [\int_{CS} r\vec{V}(\vec{V}\cdot d\vec{A})]$$

$$= \left[\text{Time change of the linear momentum within the control volume}\right]$$

$$+ \left[\text{net rate of linear momentum flux in \& out through the control surface}\right]$$

Linear Momentum

In general, there are two types of forces acting on the fluids: surface forces and body forces. Surface forces (F_s) are external forces acting on the surfaces between solids and the fluids or between different fluid layers. Examples: pressure forces, shear forces, surface tension, etc. Body forces (F_B)are acting on the entire body of the fluids, such as gravity and magnetohydrodynamic forces.

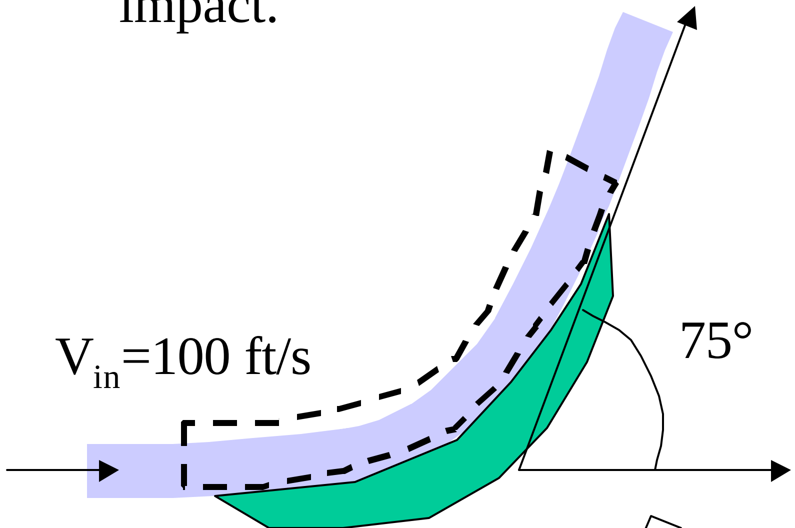
Therefore, the linear momentum equation can be written as

$$\vec{F} = \vec{F}_S + \vec{F}_B = [\frac{\partial}{\partial t}\int_{CV} r\vec{V} d\forall] + [\int_{CS} r\vec{V}(\vec{V}\cdot d\vec{A})]$$

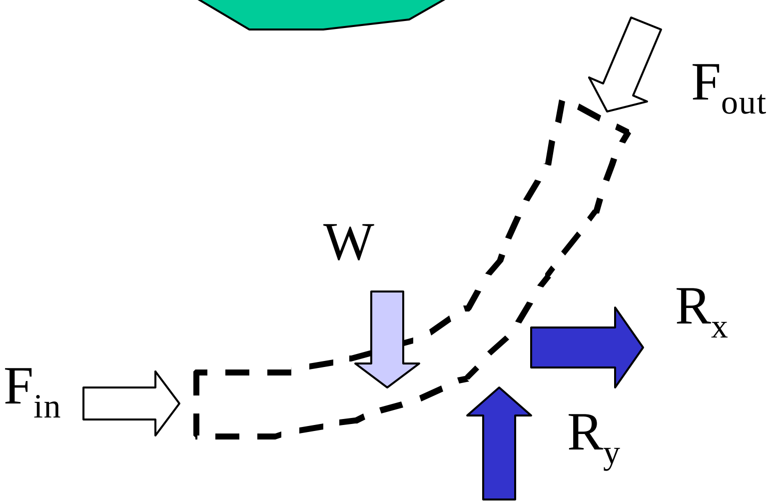
Note: These forces are forces acting on the fluids in order to produce the necessary changes of the linear momentum of the fluids. In order to evaluate the forces acting on the external devices, one needs to apply the Newton's third law: reaction force is equal in magnitude and opposite in direction with respect to the action force. That is, R = - F

Example

Consider a water jet is deflected by a stationary vane as shown. Determine the force acting on the vane by the jet if the jet speed is 100 ft/s and the diameter of the jet is 2 in. and there is no significant divergence of the jet flow during impact.



Assume steady state, shear action on the fluid does not slow down the jet significantly and the jet velocity is uniform. F_{in} & F_{out} will be static pressure acting on the jet. W is the weight of the water jet inside the CV. Assume all three contributions are also small relative to the momentum of the jet.



Therefore, the linear momentum equation can be written as

$$\vec{F}_S + \vec{F}_B = \frac{\partial}{\partial t}\int_{CV} r\vec{V}d\forall + \int_{CS} r\,\vec{V}(\vec{V}\cdot d\vec{A})$$

$$= \int_{CS} r\,\vec{V}(\vec{V}\cdot d\vec{A}), \text{ steady state}$$

Example (cont.)

The only body force is the weight of the jet, and is considered negligible, $\vec{F}_B = 0$
There are surface forces on all sides of the CV, however, contributions from three sides are also small. Therefore, only the surface forces acting on the jet by the vane are considered. These forces include the pressure force and the shear force and the sum of these forces is equal to R with components of R_x and R_y in the x and y direction, respectively.

Therefore, the linear momentum equation can be written as

$$R_x = \int_{CS} r V_x (\vec{V} \cdot d\vec{A}), \quad R_y = \int_{CS} r V_y (\vec{V} \cdot d\vec{A})$$

From mass conservation: $\frac{\partial}{\partial t} \int_{CV} r \, d\forall + \int_{CS} r \vec{V} \cdot d\vec{A} = 0$

Since no mass leaving the upper and lower boundaries, therefore

$$\int_{out} r \vec{V} \cdot d\vec{A} = r V_{out} A_{out} = -\int_{in} r \vec{V} \cdot d\vec{A} = r V_{in} A_{in} = \dot{m}$$

Example (cont.)

$$R_x = \int_{CS} r\,\mathrm{V_x}(\vec{\mathrm{V}}\cdot d\vec{\mathrm{A}}) = \int_{in}(\,) + \int_{top}(\,) + \int_{vane}(\,) + \int_{out}(\,)$$

$$= \int_{\mathrm{in}} r\,\mathrm{V_x}(\vec{\mathrm{V}}\cdot d\vec{\mathrm{A}}) + \int_{\mathrm{out}} r\,\mathrm{V_x}(\vec{\mathrm{V}}\cdot d\vec{\mathrm{A}}) = -(rV_{in}A_{in})(V_{in})_x + (rV_{out}A_{out})(V_{out})_x$$

$$= \dot{m}\left[(V_{out})_x - (V_{in})_x\right] = (1.94)(p)(1/12)^2(100)\left[(100)\cos(75°) - 100\right] = -313.5(lbf)$$

$$R_y = \int_{\mathrm{in}} r\,\mathrm{V_y}(\vec{\mathrm{V}}\cdot d\vec{\mathrm{A}}) + \int_{\mathrm{out}} r\,\mathrm{V_y}(\vec{\mathrm{V}}\cdot d\vec{\mathrm{A}}) = \dot{m}\left[(V_{out})_y - (V_{in})_y\right]$$

$$= (1.94)(p)(1/12)^2(100)\left[(100)\sin(75°) - 0\right] = 408.6(lbf)$$

The force on the vane is:

$\mathrm{K_x} = -R_x = 313.5(lbf)$, acting to the right

$K_y = -R_y = -408.6(lbf)$, acting downward