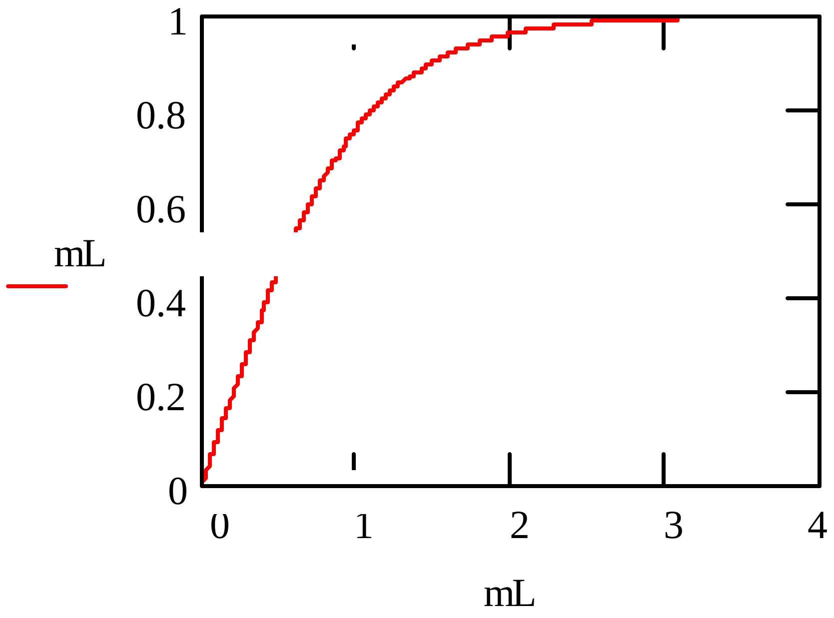
Optimal Length of a fin

In general, the longer the fin, the higher the heat transfer. However, a long fin means more material and increased size and cost. Question: how do we determine the optimal fin length? Use the rectangular fin as an example:



$q_f = M \tanh mL$, for an adiabatic tip fin

$(q_f)_\infty = M$, for an infinitely long fin

Their ratio: R(mL)= $\dfrac{q_f}{(q_f)_\infty} = \tanh mL$

Note: heat transfer increases with mL as expected. Initially the rate of change is large and slows down drastically when mL> 2.

R(1)=0.762, means any increase beyond mL=1 will increase no more than 23.8% of the fin heat transfer.

Temperature Distribution

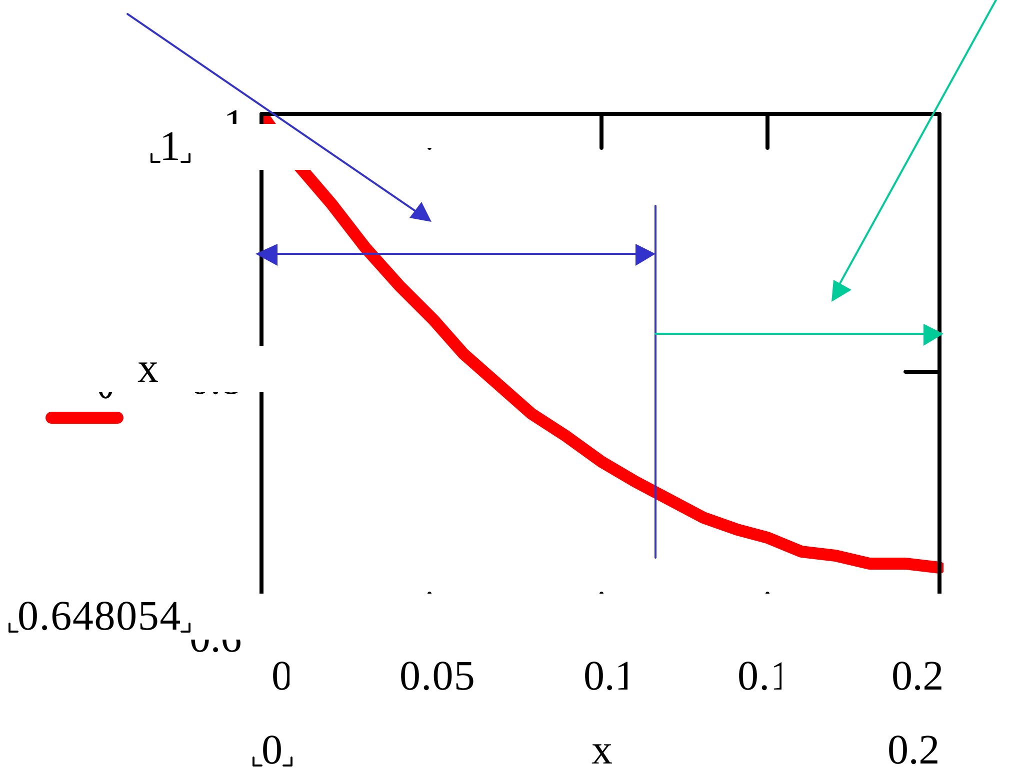
For an adiabatic tip fin case:

$$R_q = \frac{T - T_\infty}{T_b - T_\infty} = \frac{\cosh m(L-x)}{\cosh mL}$$

➢ Use m=5, and L=0.2 as an example:

Low ΔT, poor fin heat transfer

High ΔT, good fin heat transfer



Correction Length for a Fin with a Non-adiabatic Tip

The correction length can be determined by using the formula: $L_c = L + (A_c/P)$, where A_c is the cross-sectional area and P is the perimeter of the fin at the tip.

- Thin rectangular fin: $A_c = Wt$, $P = 2(W+t) \approx 2W$, since $t << W$
$L_c = L + (A_c/P) = L + (Wt/2W) = L + (t/2)$

- Cylindrical fin: $A_c = (\pi/4)D^2$, $P = \pi D$, $L_c = L + (A_c/P) = L + (D/4)$

- Square fin: $A_c = W^2$, $P = 4W$,
$L_c = L + (A_c/P) = L + (W^2/4W) = L + (W/4)$