

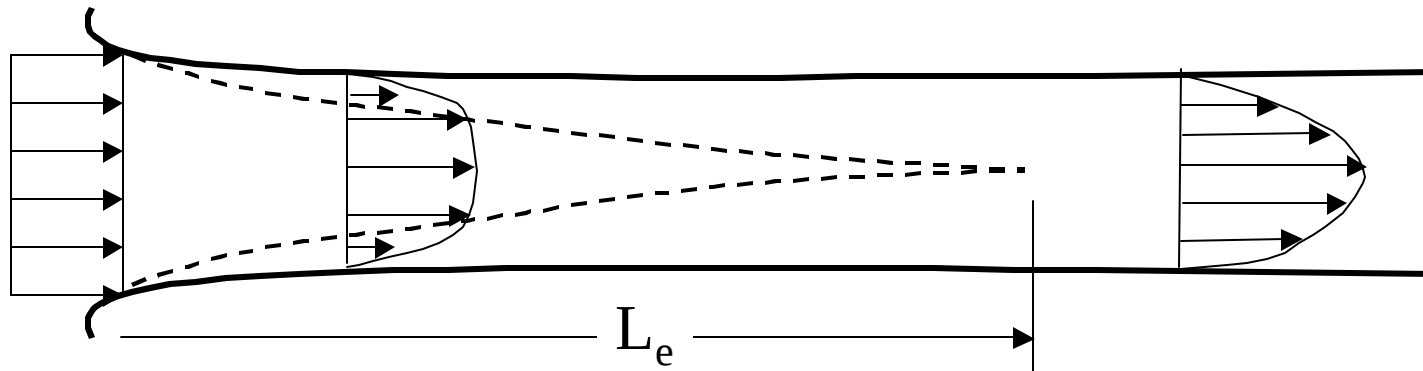
Pipe Flow Considerations

- Flow conditions:

⇒ Laminar or turbulent: transition Reynolds number $Re = \rho V D / \mu = 2,300$. That is: $Re < 2,300$ laminar; $Re > 4,000$ turbulent; $2,300 < Re < 4,000$ transitional flow.

⇒ Entrance or fully developed: flow in pipe takes some distance to adjust the sudden flow change from the pipe entrance. This distance is called the entrance length. It is a function of the Reynolds number.

- Laminar entrance length $L_e/D \approx 0.06(Re_D)$.
- Turbulent entrance length $L_e/D \approx 4.4(Re_D)^{1/6}$.



Other Considerations

Using the extended Bernoulli's equation, we have assumed previously that the velocity profile is uniform inside the pipe flow. This is obviously not true considering the fact that boundary layer flow develops inside the pipe. To take this factor into consideration, a kinetic energy coefficient, α , is defined as:

$$\int_A \frac{V^2}{2} r V dA = \alpha \int_A \frac{\bar{V}^2}{2} r V dA = \alpha m \frac{\bar{V}^2}{2}$$
$$\text{where } \alpha = \frac{\int_A r V^3 dA}{m \bar{V}^2}$$

For a laminar pipe flow, $\alpha=2.0$.

For a turbulent pipe flow:

$$\alpha = \left(\frac{U}{\bar{V}} \right)^3 \frac{2n^2}{(3+n)(3+2n)}$$

Turbulent Pipe Flow

n is the exponent in the empirical *power-law* equation of a turbulent flow in a pipe. (See equations 8.22, 8.23 & 8.24. In chapter 8.5)

$$\frac{\bar{u}(r)}{U} = \left(\frac{y}{R} \right)^{1/n} = \left(1 - \frac{r}{R} \right)^{1/n} \quad \text{where } \bar{u}(r) \text{ is the time-averaged velocity}$$

in the pipe, U is the time-averaged centerline velocity in the pipe.

$$U = \bar{u}(r = 0)$$

α is close to 1 for highly turbulent flow.

From empirical data, the exponent, n , in the power law is a function of the Reynolds number for a fully developed, smooth pipe flow: $n = -1.7 + 1.8 \log(\text{Re}_U) = -1.7 + 1.8 \log(UD/\nu)$, where U is the centerline velocity.

Modified, Extended Bernoulli's Equation

$\bar{V} = \frac{1}{A} \int_A \bar{u} dA$, therefore, according to the power law

$$\frac{\bar{V}}{U} = \frac{2n^2}{(n+1)(2n+1)} \text{ from equation 8.24}$$

$\bar{V}$ is the mean velocity of the time-averaged velocity profile $\bar{u}(r)$ averaged over the cross-section area of the pipe

Taking into consideration of the non-uniformity of the velocity profile inside a pipe flow using the kinetic energy coefficient, a , the extended Bernoulli's equation can be modified to be:

$$\frac{P_1}{\rho} + a_1 \frac{\bar{V}_1^2}{2} + gz_1 - gh_L = \frac{P_2}{\rho} + a_2 \frac{\bar{V}_2^2}{2} + gz_2$$

Non-circular Pipes

If the cross section of a pipe is not circular, then all previous relations can not be used directly. A new parameter, hydraulic diameter, is defined to correlate the flow in a non-circular pipes to that of a circular pipe flow. This correlation can be applied to many different pipe configurations as long as their cross sectional shapes are not too exaggerated. For example, width to height ratio is less than 4 for a rectangular pipe. The hydraulic diameter is defined as:

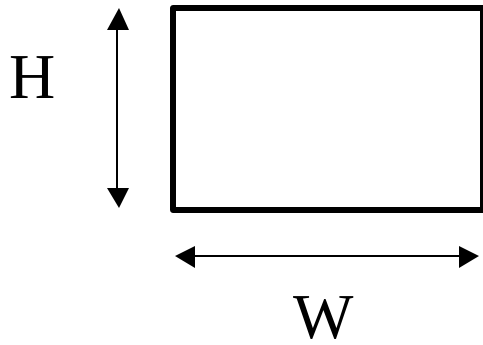
$$D_h = \frac{4A}{P}, \text{ where } A \text{ is the cross-sectional area,}$$

and P is the wetted perimeter, or the total length of the surface which is in contact with the fluid in one cross section.

Example

For a rectangular duct with width W and height H , the hydraulic diameter is

$$D_h = \frac{4A}{P} = \frac{4WH}{2(W + H)} = \frac{2WH}{W + H}$$



In order to use the circular pipe flow relationship, we need to use the hydraulic diameter, D_h , to replace the pipe diameter, D , used in the circular pipe flow problem.