Frictional Losses in a Pipe Flow (Major Losses)

<u>Frictional Losses:</u>

From the dimensional analysis, we learned that the normalized pressure drop, $\Delta P/(½\rho V^2)$ in a pipe flow due to the frictional loss can be related to other dimensionless parameters such as the pipe Reynolds number ($\rho VD/\mu$), relative roughness (ε/D) and the length to diameter ratio (L/D). That is, $\dfrac{\Delta P}{\frac{1}{2} rV^2} = f_1(\dfrac{rVD}{m}, \dfrac{L}{D}, \dfrac{e}{D})$.

It is logical to assume that the pressure drop is linearly proportional to the pipe length so that

$$\frac{\Delta P}{\frac{1}{2} rV^2} = \frac{L}{D} f_2(\frac{rVD}{m}, \frac{e}{D}),$$

or $\Delta P = \dfrac{L}{D}(\frac{1}{2} rV^2) f_2(\dfrac{rUD}{m}, \dfrac{e}{D}) = f\,\dfrac{L}{D}(\frac{1}{2} rV^2)$

by defining the frictional factor $f = f_2(\dfrac{rVD}{m}, \dfrac{e}{D}) = \dfrac{\Delta P}{\frac{1}{2} rV^2}(\dfrac{D}{L})$.

Frictional Factor

The frictional factor is then a function of the Reynolds number and the relative roughness alone. For a laminar flow (Re<2,300) inside a horizontal pipe, the frictional factor is not correlated to the relative roughness and is a function of the Reynolds number alone. It can be shown that the frictional factor is simply f = 64/Re (see chapter 8.2 in the FM book for the derivation). If the pipe flow is turbulent (Re>4,000) its frictional factor relation can not be determined analytically and it is usually determined empirically and is tabulated in tables or charts (see the Moody chart in chapter 8.4 in the text). The frictional factor is plotted as a function of the Reynolds number and relative roughness. Typical roughness values for commercially available pipes are listed in Table 8.1 as a reference. Other frictional factor correlation formulas are also available. The most famous one is the Colebrook equation:

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{e/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right).$$

The drawback of using this equation is that f is not explicitly related to Re and ε/D. Sometimes, the following equation can be used instead:

$$f = \frac{1.325}{\left\{\ln\left[\left(e/3.7D\right) + \left(5.74/\mathrm{Re}^{0.9}\right)\right]\right\}^2} \text{ for } 10^{-6} < e/D < 10^{-2} \text{ and } 5000<\mathrm{Re}<10^8.$$

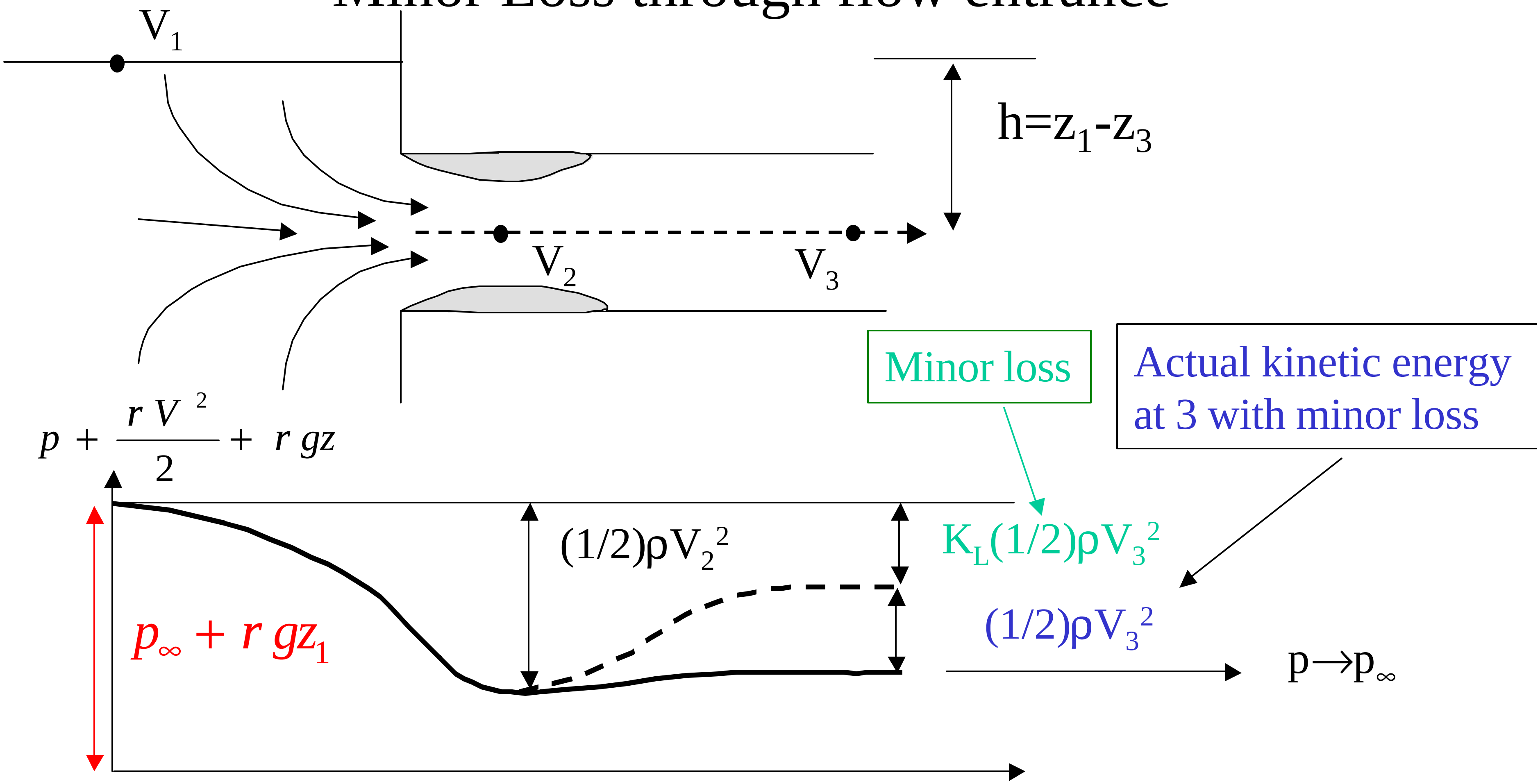
Minor Losses

Frictional losses (major losses) usually are responsible for the majority of the pressure losses in a pipe system. However, pipe systems usually consist of many other components, such as valves, bends, elbows, expansions, etc., which also contribute to the total head loss of the system. These losses can be significant if the pipe length is not very long. Discussion about the minor losses can be found in chapter 8.4.2 in the textbook. The pressure drop due to minor losses can be specified by using the loss coefficient, K_L, which is defined as

$$K_L = \frac{\Delta P}{\frac{1}{2} r V^2} \text{ so that } \Delta P = K_L \left(\frac{1}{2} r V^2\right).$$

One of the example of minor losses is the entrance flow loss. A typical flow pattern for flow entering a sharp-edged entrance is shown in the following page. A vena contracta region if formed at the inlet because the fluid can not turn a sharp corner. Flow separation and associated viscous effects will tend to decrease the flow energy and the phenomenon is complicated. To simplify the analysis, a head loss and the associated loss coefficient are used in the extended Bernoulli's equation to take into consideration of this effect as described in the next page.

Minor Loss through flow entrance



Extended Bernoulli's Equation: $\dfrac{p_1}{g} + \dfrac{V_1^2}{2g} + z_1 - h_L = \dfrac{p_3}{g} + \dfrac{V_3^2}{2g} + z_3, \quad h_L = K_L \dfrac{V_3^2}{2g}$

$$p_1 = p_3 = p_\infty, \quad V_1 \approx 0, \quad V_3 = \frac{1}{\sqrt{1+K_L}} \left(\sqrt{2g(z_1 - z_3)}\right) = \sqrt{\frac{2}{1+K_L} gh}$$

Extended Bernoulli's Equation

Based on the previous discussion, the pressure distribution along a pipe system can be characterized by using the extended Bernoulli's equation:

$$\frac{P_1}{r} + \frac{V_1^2}{2} + gz_1 + gh_A - gh_E - gh_L = \frac{P_2}{r} + \frac{V_2^2}{2} + gz_2$$

For a horizontal(z_1=z_2), fully-developed(V_1=V_2), straight pipe system with no external po

(h_A=h_E=0), the pressure drop is related to the losses alone, that is,

$$P_1 - P_2 = \Delta P = r\,gh_L = \text{frictional losses + minor losses} = \text{f}\,\frac{\text{L}}{\text{D}}\left(\tfrac{1}{2} rV^2\right) + K_L\left(\tfrac{1}{2} rV^2\right)$$

Both the loss coefficient and the frictional factor can be determined using the empirical values available in standardized tables or charts in chapter 8.

Special note: due to the non-uniformity of the velocity profiles inside a pipe, the kinetic energy terms in the extended Bernoulli's equation should be modified to include this effect. The kinetic energy coefficient, a, is so defined in chapter 8.6.1. See equations 8.28 and 8.29 to get more information.