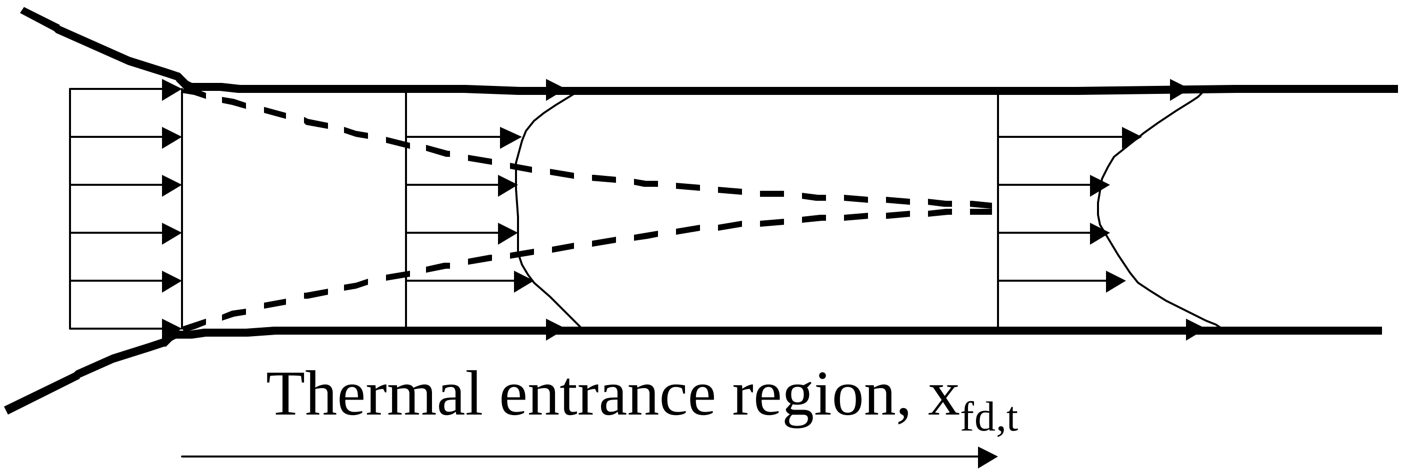
Thermal Considerations in a Pipe Flow

- Thermal conditions
  - ⇨ Laminar or turbulent
  - ⇨ entrance flow and fully developed thermal condition

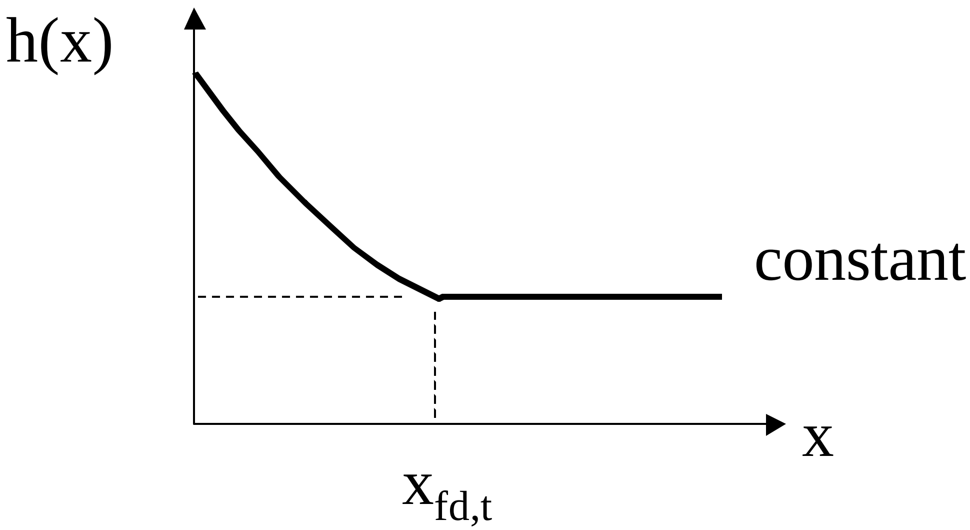


For laminar flows the thermal entrance length is a function of the Reynolds number and the Prandtle number: $x_{fd,t}/D \approx 0.05 Re_D Pr$, where the Prandtl number is defined as $Pr = \nu/\alpha$ and α is the thermal diffusivity.

For turbulent flow, $x_{fd,t} \approx 10D$.

Thermal Conditions

• For a fully developed pipe flow, the convection coefficient is a constant and is not varied along the pipe length. (as long as all thermal and flow properties are constant also.)



• Newton's law of cooling: $q''_S = hA(T_S - T_m)$

Question: since the temperature inside a pipe flow is not constant, what temperature we should use. A mean temperature T_m is defined.

Energy Transfer

Consider the total thermal energy carried by the fluid as

$$\int_A rVC_v TdA = \text{(mass flux) (internal energy)}$$

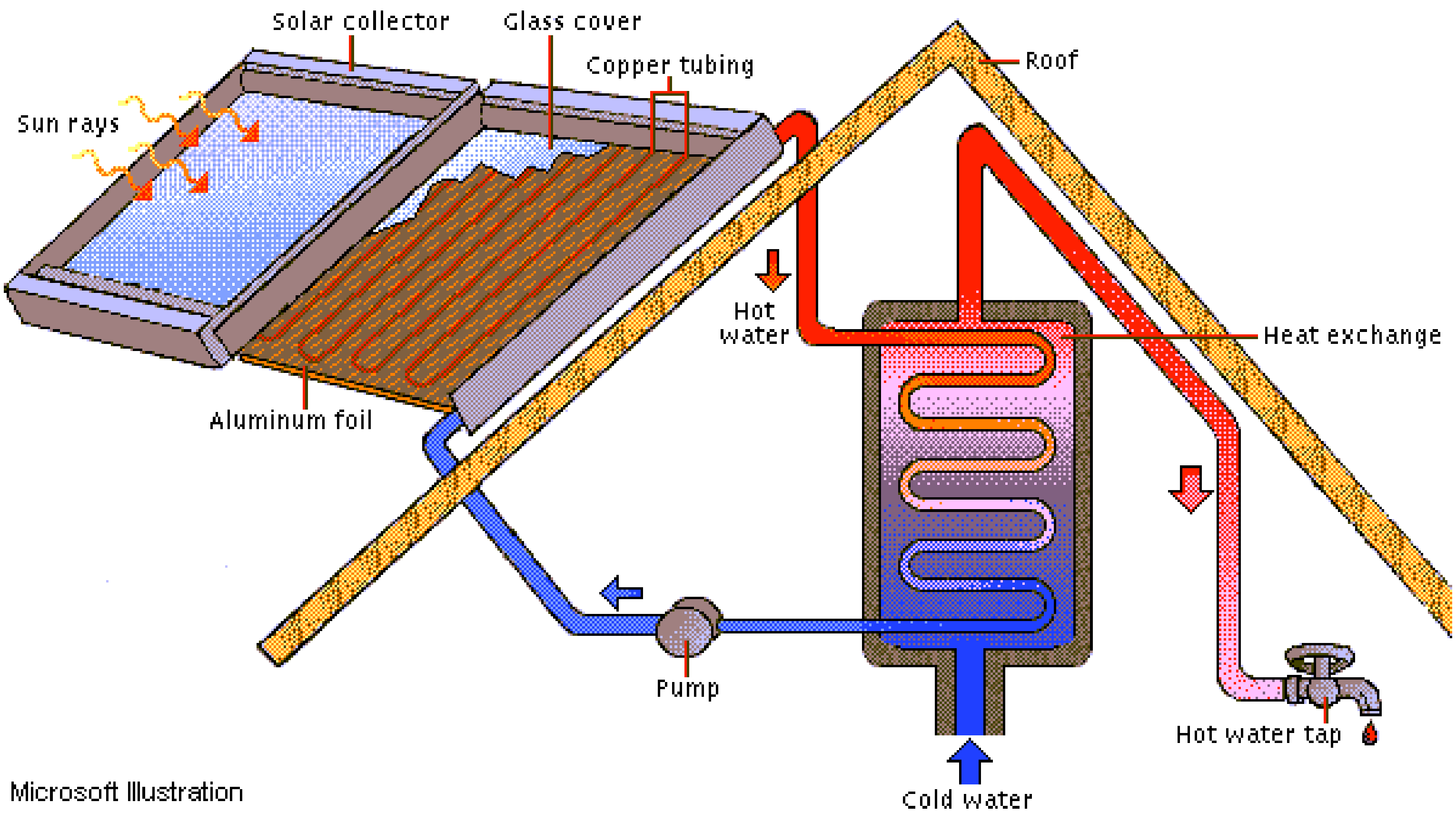
Now image this same amount of energy is carried by a body of fluid with the same mass flow rate but at a uniform mean temperature T_m. Therefore T_m can be defined as

$$T_m = \frac{\int_A rVC_v TdA}{\dot{m}C_v}$$

Consider T_m as the reference temperature of the fluid so that the total heat transfer between the pipe and the fluid is governed by the Newton's cooling law as: q_s"=h(T_s-T_m), where h is the local convection coefficient, and T_s is the local surface temperature. Note: usually T_m is not a constant and it varies along the pipe depending on the condition of the heat transfer.

Energy Balance

Example: We would like to design a solar water heater that can heat up the water temperature from 20° C to 50° C at a water flow rate of 0.15 kg/s. The water is flowing through a 5 cm diameter pipe and is receiving a net solar radiation flux of 200 W per unit length (meter). Determine the total pipe length required to achieve the goal.



Example (cont.)

Questions: (1) How do we determine the heat transfer coefficient, h?

There are a total of six parameters involving in this problem: h, V, D, ν, k_f, c_p. The last two variables are thermal conductivity and the specific heat of the water. The temperature dependence is implicit and is only through the variation of thermal properties. Density ρ is included in the kinematic viscosity, $\nu=\mu/\rho$. According to the Buckingham theorem, it is possible for us to reduce the number of parameters by three. Therefore, the convection coefficient relationship can be reduced to a function of only three variables:

$Nu=hD/k_f$, Nusselt number, $Re=VD/\nu$, Reynolds number, and
$Pr=\nu/\alpha$, Prandtle number.

This conclusion is consistent with empirical observation, that is $Nu=f(Re, Pr)$. If we can determine the Reynolds and the Prandtle numbers, we can find the Nusselt number, hence, the heat transfer coefficient, h.

Convection Correlations

⇒ Laminar, fully developed circular pipe flow:

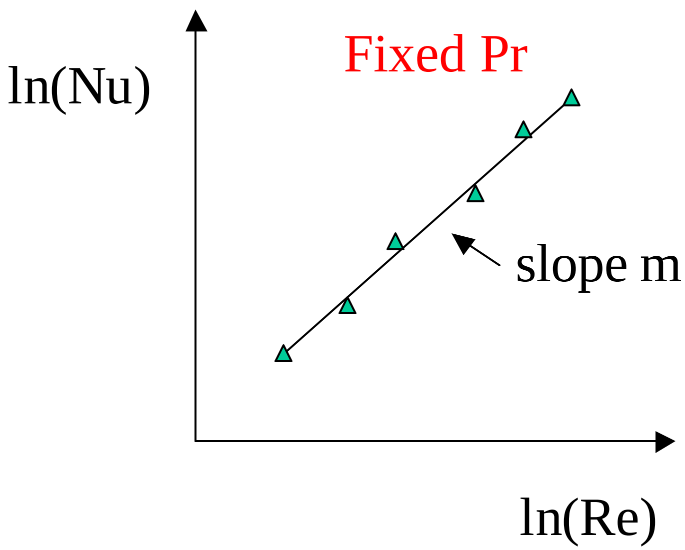
$$Nu_D = \frac{hD}{k_f} = 4.36, \quad \text{when } q_s'' = \text{constant, (page 543, ch. 10-6, ITHT)}$$

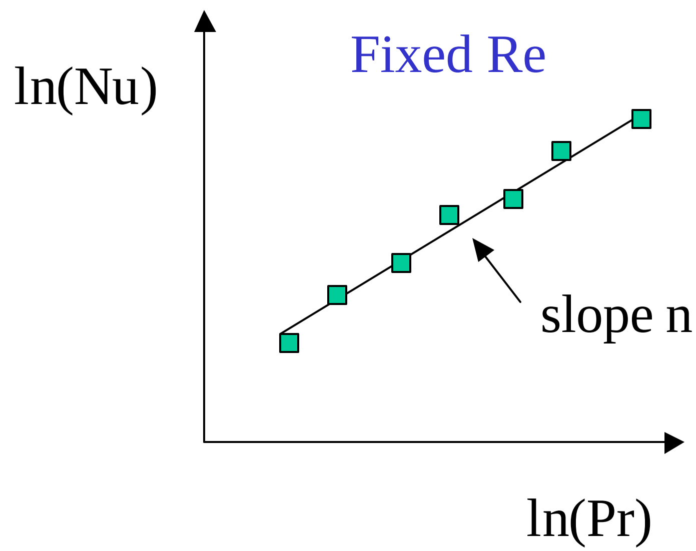
$$Nu_D = 3.66, \quad \text{when } T_s = \text{constant, (page 543, ch. 10-6, ITHT)}$$

Note: the therma conductivity should be calculated at T_m.

⇒ Fully developed, turbulent pipe flow: Nu = f(Re, Pr),

Nu can be related to Re & Pr experimentally, as shown.





Empirical Correlations

Dittus-Boelter equation: $\mathrm{Nu_D} = 0.023\mathrm{Re}^{4/5}\,\mathrm{Pr}^{n}$, (eq 10-76, p 546, ITHT)

where n = 0.4 for heating ($T_s > T_m$), n = 0.3 for cooling ($T_s < T_m$).

The range of validity: $0.7 \le \mathrm{Pr} \le 160$, $\mathrm{Re_D} \ge 10{,}000$, $L/D \ge 10$.

Note: This equation can be used only for moderate temperature difference with all the properties evaluated at T_m.

Other more accurate correlation equations can be found in other references.
Caution: The ranges of application for these correlations can be quite different.

For example, the Gnielinski correlation is the most accurate among all these equations:

$$\mathrm{Nu_D} = \frac{(f/8)(\mathrm{Re}_D - 1000)\,\mathrm{Pr}}{1 + 12.7(f/8)^{1/2}(\mathrm{Pr}^{2/3} - 1)} \quad \text{(from other reference)}$$

It is valid for $0.5 < \mathrm{Pr} < 2000$ and $3000 < \mathrm{Re_D} < 5 \times 10^6$.

All properties are calculated at T_m.

Example (cont.)

In our example, we need to first calculate the Reynolds number: water at 35°C, C_p=4.18(kJ/kg.K), μ=7x10^{-4} (N.s/m^2), k_f=0.626 (W/m.K), Pr=4.8.

$$\mathrm{Re} = \frac{rVD}{m} = \frac{\dot{m}/A\, D}{m} = \frac{4\dot{m}}{p\,Dm} = \frac{4(0.15)}{p\,(0.05)(7\times 10^{-4})} = 5460$$

Re > 4000, it is turbulent pipe flow.

Use the Gnielinski correlation, from the Moody chart, f = 0.036, Pr = 4.8

$$\mathrm{Nu_D} = \frac{(f/8)(\mathrm{Re}_D - 1000)\,\mathrm{Pr}}{1 + 12.7(f/8)^{1/2}(\mathrm{Pr}^{2/3} - 1)} = \frac{(0.036/8)(5460 - 1000)(4.8)}{1 + 12.7(0.036/8)^{1/2}(4.8^{2/3} - 1)} = 37.4$$

$$h = \frac{k_f}{D} Nu_D = \frac{0.626}{0.05}(37.4) = 469(W/m^2.K)$$

Energy Balance

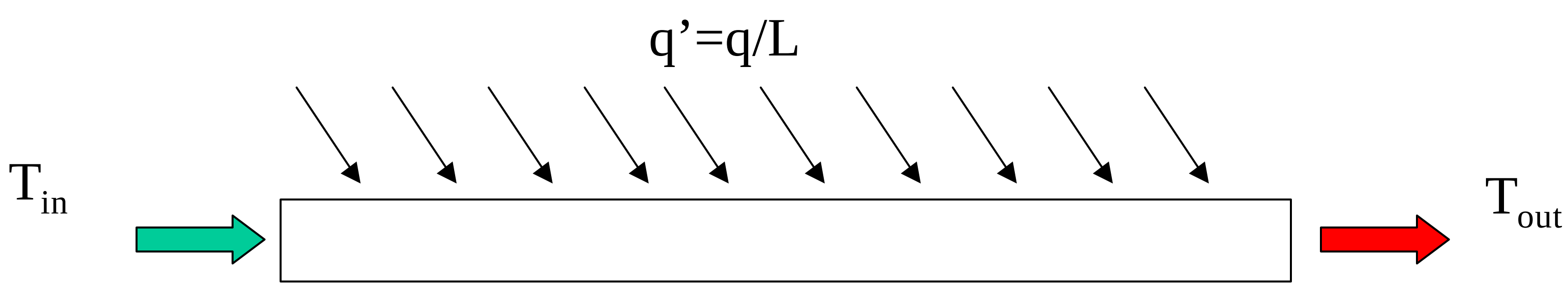
Question (2): How can we determine the required pipe length?

Use energy balance concept: (energy storage) = (energy in) minus (energy out).

energy in = energy received during a steady state operation (assume no loss)

$$q'(L) = \dot{m}C_P(T_{out} - T_{in}),$$

$$L = \frac{\dot{m}C_P(T_{in} - T_{out})}{q'} = \frac{(0.15)(4180)(50-20)}{200} = 94(m)$$



Temperature Distribution

Question (3): Can we determine the water temperature variation along the pipe?

Recognize the fact that the energy balance equation is valid for any pipe length x:

$$q'(x) = \dot{m}C_P(T(x) - T_{in})$$

$$T(x) = T_{in} + \frac{q'}{\dot{m}C_P}x = 20 + \frac{200}{(0.15)(4180)}x = 20 + 0.319x$$

It is a linear distribution along the pipe

Question (4): How about the surface temperature distribution?

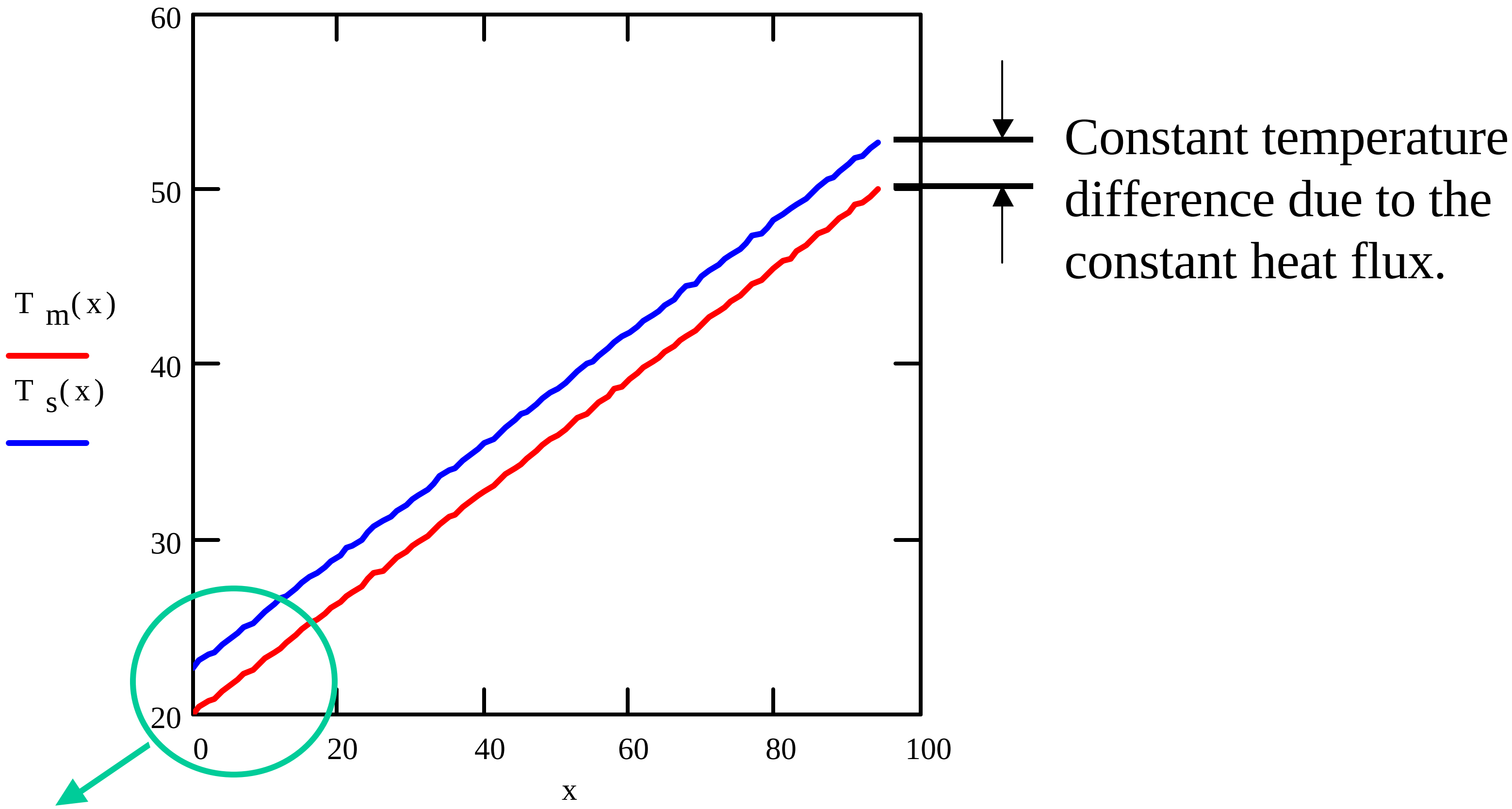
From local Newton's cooling law:

$$\mathrm{q} = \mathrm{hA}(\mathrm{T_s} - T_m) \Rightarrow q'\Delta x = h(p\,D\Delta x)(\mathrm{T_s}(x) - T_m(x))$$

$$T_s(x) = \frac{q'}{p\,Dh} + T_m(x) = \frac{200}{p(0.05)(469)} + 20 + 0.319x = 22.7 + 0.319x \quad (°C)$$

At the end of the pipe, $\mathrm{T_s}(x = 94) = 52.7(°C)$

Temperature variation for constant heat flux



Note: These distributions are valid only in the fully developed region. In the entrance region, the convection condition should be different. In general, the entrance length x/D≈10 for a turbulent pipe flow and is usually negligible as compared to the total pipe length.