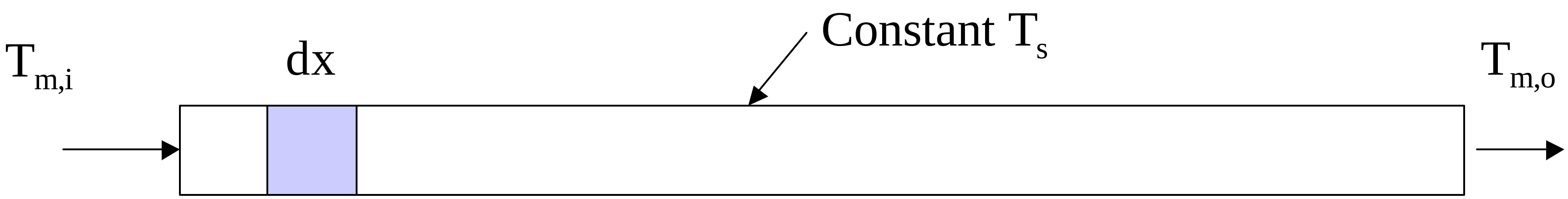
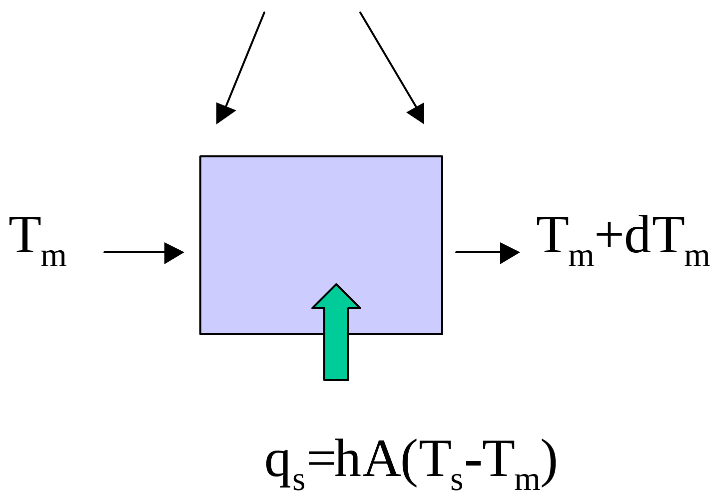
Internal Flow Convection
-constant surface temperature case

Another commonly encountered internal convection condition is when the surface temperature of the pipe is a constant. The temperature distribution in this case is drastically different from that of a constant heat flux case. Consider the following pipe flow configuration:





$$\text{Energy change} = \dot{m}C_p[(T_m + dT_m) - T_m]$$

$$= \dot{m}C_p dT_m$$

$$\text{Energy in} = hA(T_s - T_m)$$

$$\text{Energy change} = \text{energy in}$$

$$\dot{m}C_p dT_m = hA(T_s - T_m)$$

Temperature distribution

$\dot{m}C_p dT_m = hA(T_s - T_m),$

Note: $q = hA(T_s - T_m)$ is valid locally only, since T_m is not a constant

$$\frac{dT_m}{(T_m - T_s)} = -\frac{hA}{\dot{m}C_P},$$ where A = Pdx, and P is the perimeter of the pipe

Integrate from the inlet to a diatance x downstream:

$$\int_{T_{m,i}}^{T_m(x)} \frac{dT_m}{(T_m - T_s)} = -\int_0^x \frac{hP}{\dot{m}C_P} dx = -\frac{P}{\dot{m}C_P}\int_0^x h dx$$

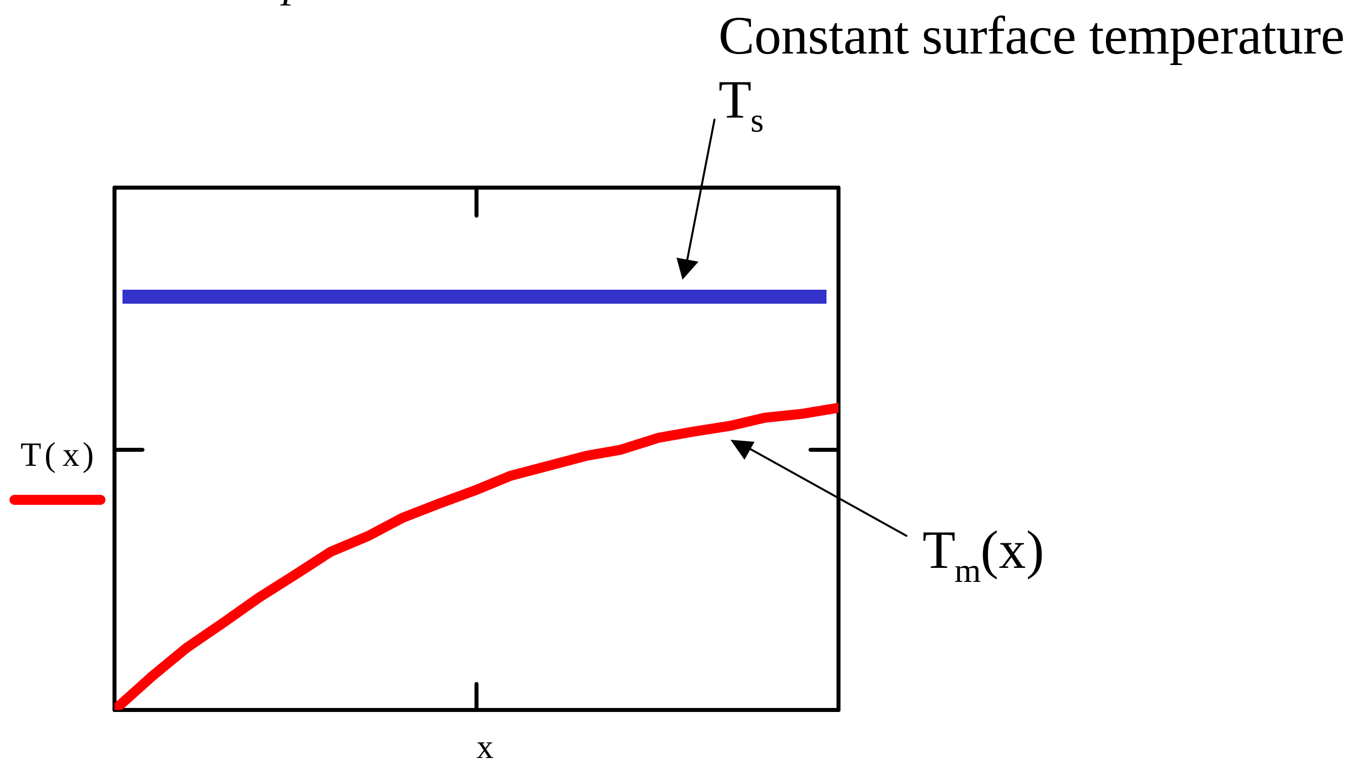
$$\ln(T_m - T_s)\Big|_{T_{m,i}}^{T_m(x)} = -\frac{P\bar{h}}{\dot{m}C_P} x,$$ where L is the total pipe length

and $\bar{h}$ is the averaged convection coefficient of the pipe between 0 & x.

$$\bar{h} = \frac{1}{x}\int_0^x h dx, \qquad \text{or } \int_0^x h dx = \bar{h}x$$

Temperature distribution

$$\frac{T_m(x) - T_s}{T_{m,i} - T_s} = \exp\left(-\frac{P\bar{h}}{\dot{m}C_P}x\right), \text{ for constant surface temperature}$$



The difference between the averaged fluid temperature and the surface temperature decreases exponentially further downstream along the pipe.

Log-Mean Temperature Difference

For the entire pipe:

$$\frac{T_{m,o}-T_s}{T_{m,i}-T_s}=\frac{\Delta T_o}{\Delta T_i}=\exp\left(-\frac{\bar{h}(PL)}{\dot{m}C_P}\right) \quad \text{or } \dot{m}C_P=-\frac{\bar{h}A_s}{\ln\left(\dfrac{\Delta T_o}{\Delta T_i}\right)}$$

$$q=\dot{m}C_P(T_{m,o}-T_{m,i})=\dot{m}C_P((T_s-T_{m,i})-(T_s-T_{m,o}))$$

$$=\dot{m}C_P(\Delta T_i-\Delta T_o)=\bar{h}A_s\frac{\Delta T_o-\Delta T_i}{\ln\left(\dfrac{\Delta T_o}{\Delta T_i}\right)}=\bar{h}A_s\Delta T_{lm}$$

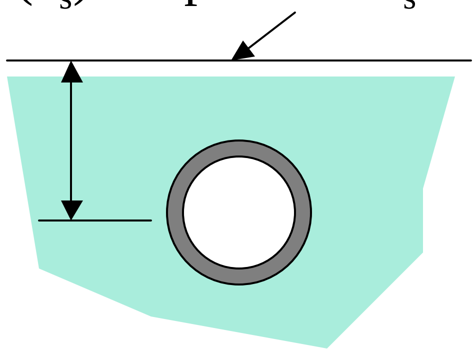
where $\Delta T_{lm}=\dfrac{\Delta T_o-\Delta T_i}{\ln\left(\dfrac{\Delta T_o}{\Delta T_i}\right)}$ is called the log mean temperature difference.

This relation is valid for the entire pipe.

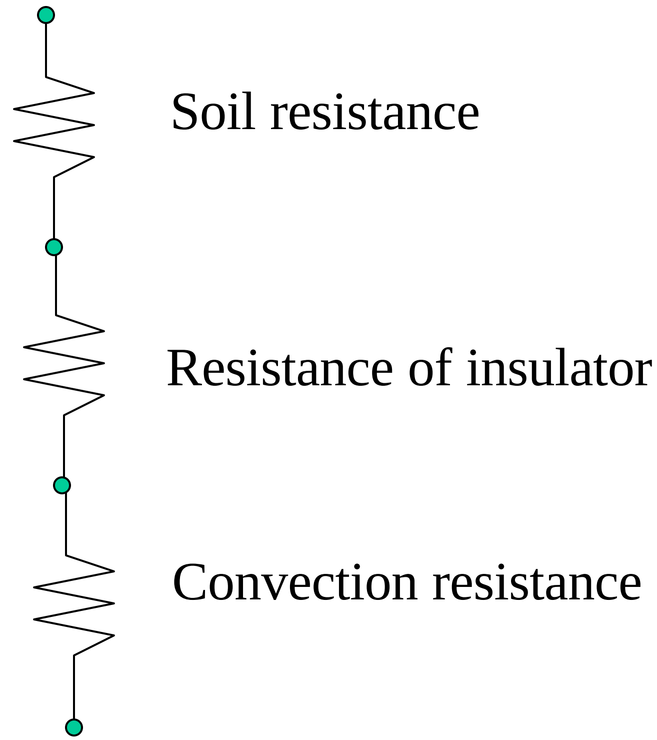
External Heat Transfer

Can we extend the previous analysis to include the situation that some external heat transfer conditions are given, rather than that the surface temperature is given. Example: Pipe flow buried underground with insulation. In that case, the heat transfer is first from the fluid to the pipe wall through convection; then followed by the conduction through the insulation layer; finally, heat is transferred to the soil surface by conduction. See the following figure:

Soil (k_s) temperature T_s



Diameter D, insulation thickness t



Overall Heat Transfer Coefficient

From the previous example, the total thermal resistance can be written as $R_{total}=R_{soil}+R_{insulator}+R_{convection.}$

The heat transfer can be expressed as: $q=\Delta T_{lm}/R_{tot}=UA_s\ \Delta T_{lm}$
by defining the overall heat transfer coefficient $UA_s=1/R_{tot}$. (Consider U as an equivalent heat transfer coefficient taking into consideration of all heat transfer modes between two constant temperature sources.)

We can replace the convection coefficient h by U in the temperature distribution equation derived earlier:

$$\frac{T_{m,o}-T_{soil}}{T_{m,i}-T_{soil}}=\frac{\Delta T_o}{\Delta T_i}=\exp\left(-\frac{UA_s}{\dot{m}C_P}\right)=\exp\left(-\frac{1}{\dot{m}C_P R_{tot}}\right)$$