Example

Determine the boundary layer thickness, the wall shear stress of a laminar water flow over a flat plate. The freestream velocity is 1 m/s, the kinematic viscosity of the water is 10^{-6} m^2/s. The density of the water is 1,000 kg/m^3. The transition Reynolds number $Re=Ux/\nu=5\times10^5$. Determine the distance downstream of the leading edge when the boundary transitions to turbulent. Determine the total frictional drag produced by the laminar portion of the plate.

$$d(x) = 5\sqrt{\frac{n x}{U_\infty}} = 5\times10^{-3}\sqrt{x} \quad (m).$$

Therefore, for a 1m long plate, the boundary layer grows by 0.005(m), or 5 mm, a very thin layer.

The wall shear stress, $t_w = \dfrac{0.332 r U_\infty^2}{\sqrt{\mathrm{Re}_x}} = 0.332 U_\infty \sqrt{\dfrac{r m U_\infty}{x}} = \dfrac{0.0105}{\sqrt{x}} (Pa)$

The transition Reynolds number: $\mathrm{Re} = \dfrac{\mathrm{U}_\infty x_{tr}}{n} = 5\times10^5, \quad x_{tr} = 0.5(m)$

Example (cont.)

The total frictional drag is equal to the integration of the wall shear stress:

$$\mathrm{F_D} = \int_0^{x_{tr}} t_w (1) dx = \int_0^{x_{tr}} 0.332 U_\infty \sqrt{\frac{r m U_\infty}{x}} dx = \frac{0.664 r U_\infty^2}{\sqrt{\mathrm{Re}_{x_{tr}}}} = 0.939(N)$$

Define skin friction coefficient: $\mathrm{C_f}$

$$C_f = \frac{t_w}{\frac{1}{2} r U_\infty^2} = \frac{0.664}{\sqrt{\mathrm{Re}_x}} \text{ for a laminar boundary layer.}$$