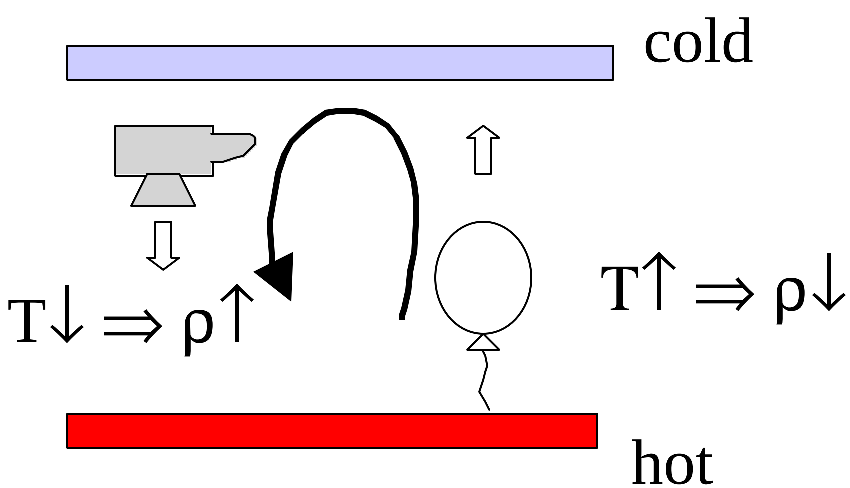
Free Convection

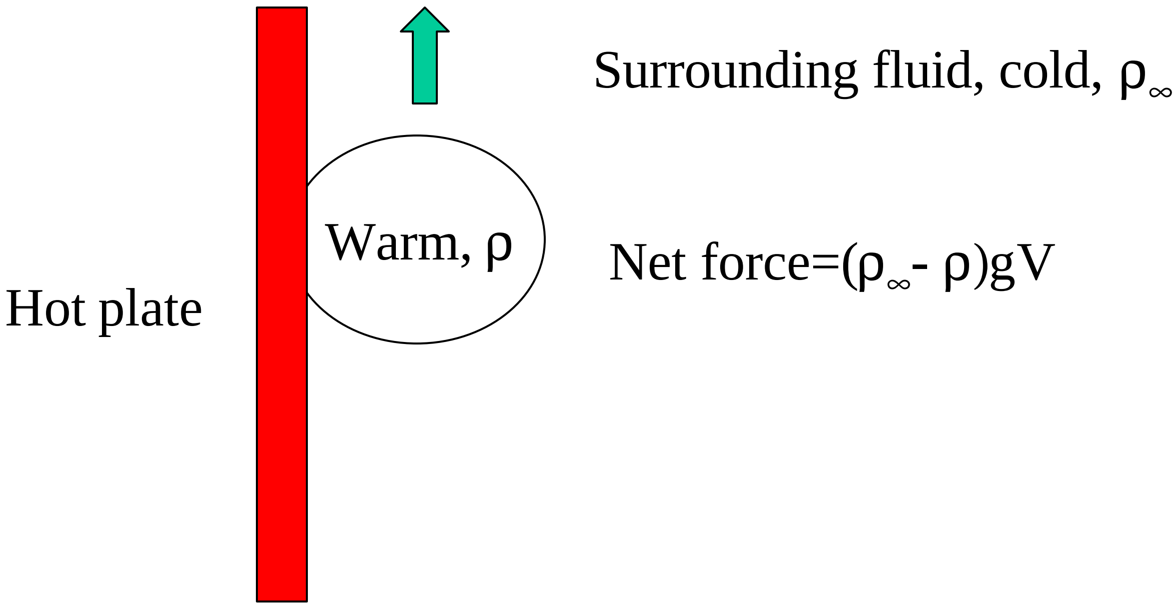
A free convection flow field is a self-sustained flow driven by the presence of a temperature gradient. (As opposed to a forced convection flow where external means are used to provide the flow.) As a result of the temperature difference, the density field is not uniform also. Buoyancy will induce a flow current due to the gravitational field and the variation in the density field. In general, a free convection heat transfer is usually much smaller compared to a forced convection heat transfer. It is therefore important only when there is no external flow exists.



Flow is unstable and a circulatory pattern will be induced.

Basic Definitions

Buoyancy effect:



The density difference is due to the temperature difference and it can be characterized by ther volumetric thermal expansion coefficient, β:

$$b = -\frac{1}{r}\left(\frac{\partial r}{\partial T}\right)_P \approx -\frac{1}{r}\frac{r_\infty - r}{T_\infty - T} = -\frac{1}{r}\frac{\Delta r}{\Delta T}$$

$$\Delta r \approx b\Delta T$$

Grashof Number and Rayleigh Number

Define Grashof number, Gr, as the ratio between the buoyancy force and the viscous force:

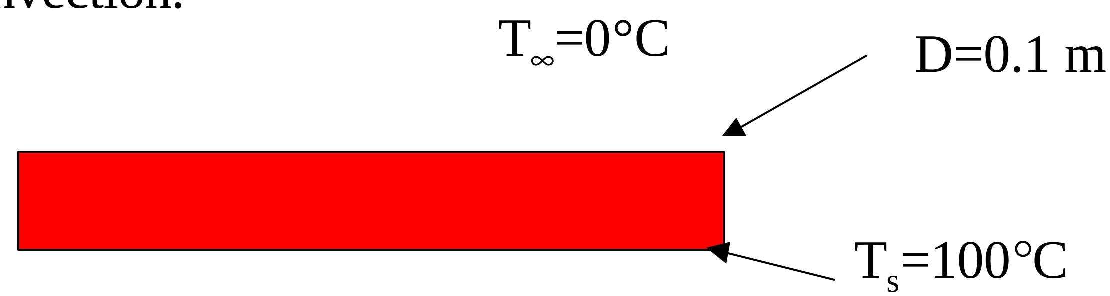
$$Gr = \frac{g\beta\Delta T L^3}{\nu^2} = \frac{g\beta(T_S - T_\infty)L^3}{\nu^2}$$

• Grashof number replaces the Reynolds number in the convection correlation equation. In free convection, buoyancy driven flow sometimes dominates the flow inertia, therefore, the Nusselt number is a function of the Grashof number and the Prandtle number alone. Nu=f(Gr, Pr). Reynolds number will be important if there is an external flow. (see chapter 11.5, combined forced and free convection.

• In many instances, it is better to combine the Grashof number and the Prandtle number to define a new parameter, the Rayleigh number, Ra=GrPr. The most important use of the Rayleigh number is to characterize the laminar to turbulence transition of a free convection boundary layer flow. For example, when $Ra>10^9$, the vertical free convection boundary layer flow over a flat plate becomes turbulent.

Example

Determine the rate of heat loss from a heated pipe as a result of natural (free) convection.



Film temperature(T_f): averaged boundary layer temperature T_f=1/2(T_s+T_∞)=50 °C.
k_f=0.03 W/m.K, Pr=0.7, ν=2×10^{-5} m^2/s, β=1/T_f=1/(273+50)=0.0031(1/K)

$$Ra = \frac{g\,b\,(T_S - T_\infty)L^3}{n^2}\,\mathrm{Pr} = \frac{(9.8)(0.0031)(100-0)(0.1)^3}{(2\times10^{-5})^2}(0.7) = 7.6\times10^6.$$

$$Nu_D = \{0.6 + \frac{0.387\,Ra^{1/6}}{[1+(0.559/\mathrm{Pr})^{9/16}]^{8/27}}\}^2 = 26.0 \quad \text{(equation 11.15 in Table 11.1)}$$

$$h = \frac{k_f}{D}Nu_D = \frac{0.03}{0.1}(26) = 7.8(W/m^2K)$$

$$q = hA(T_S - T_\infty) = (7.8)(p)(0.1)(1)(100-0) = 244.9(W)$$

Can be significant if the pipe are long.