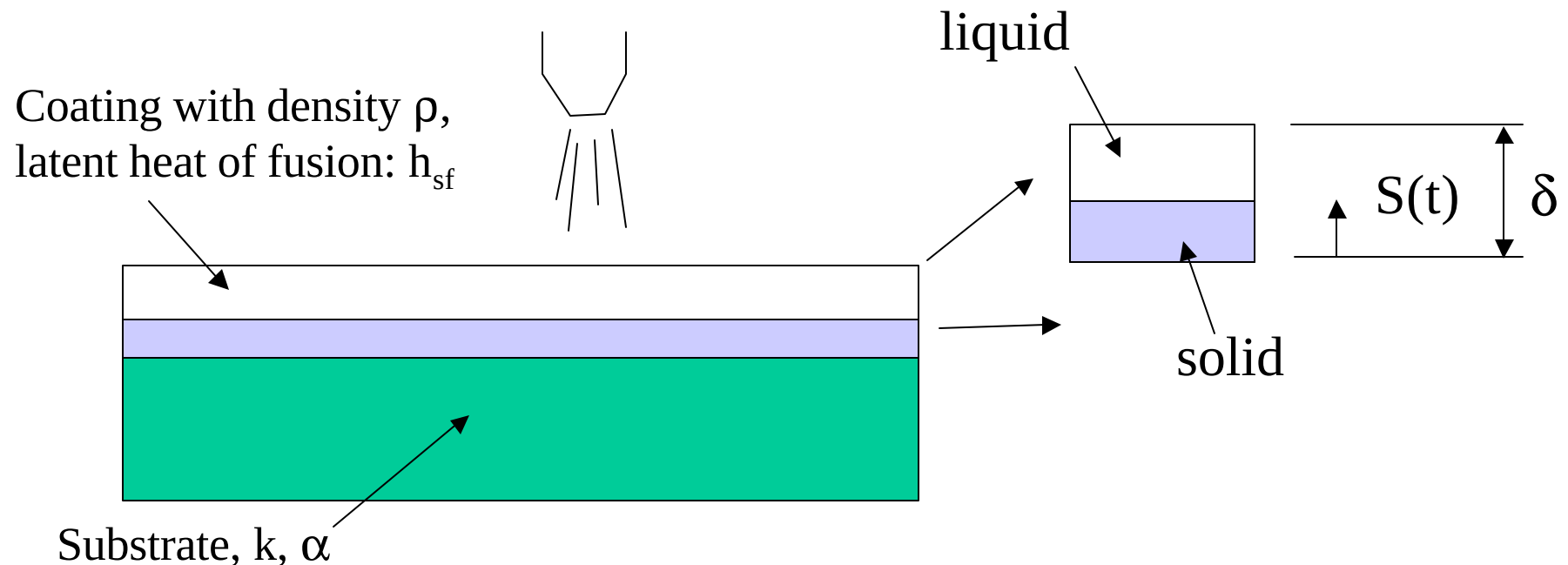


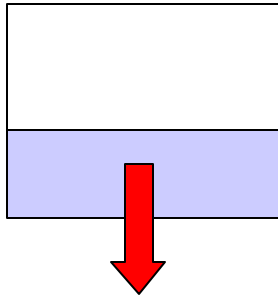
Unsteady Heat Transfer in Semi-infinite Solids

Solidification process of the coating layer during a thermal spray operation is an unsteady heat transfer problem. As we discuss earlier, thermal spray process deposits thin layer of coating materials on surface for protection and thermal resistant purposes, as shown. The heated, molten materials will attach to the substrate and cool down rapidly. The cooling process is important to prevent the accumulation of residual thermal stresses in the coating layer.



Example

As described in the previous slide, the cooling process can now be modeled as heat loss through a semi-infinite solid. (Since the substrate is significantly thicker than the coating layer) The molten material is at the fusion temperature T_f and the substrate is maintained at a constant temperature T_i . Derive an expression for the total time that is required to solidify the coating layer of thickness δ .



Heat transfer from
the molten material
to the substrate
($q=q''A$)

- Assume the molten layer stays at a constant temperature T_f throughout the process. The heat loss to the substrate is solely supplied by the release of the latent heat of fusion.

From energy balance:

$$h_{sf} \Delta m (\text{solidified mass during } \Delta t) = \Delta Q = q'' A \Delta t (\text{energy input})$$

$$h_{sf} \frac{dm}{dt} = q'' A, \text{ where } m = rV = rAS,$$

where S is solidified thickness

$$r \frac{dS}{dt} = q''$$

Example (cont.)

Identify that the previous situation corresponds to the case discussed in chapter 9-3 in the text as a semi-infinite transient heat transfer problem with a constant surface temperature boundary condition (note: the case in the textbook corresponds to an external convection case. However, it can be modelled as constant surface temperature case by setting $h=\infty$, therefore, $T_s=T_\infty$).

If the surface temperature is T_s and the initial temperature of the block is T_i , the analytical solution of the problem can be found:
The temperature distribution and the heat transfer into the block are:

$$\frac{T(x,t)-T_s}{T_i-T_s} = \operatorname{erf}\left(\frac{x}{2\sqrt{at}}\right), \text{ where } \operatorname{erf}(\) \text{ is the Gaussian error function.}$$

$$\text{It is defined as } \operatorname{erf}(w) = \frac{2}{\sqrt{\pi}} \int_0^w e^{-v^2} dv$$

$$q_s''(t) = \frac{k(T_s - T_i)}{\sqrt{\pi at}}$$

Example (cont.)

From the previous equation

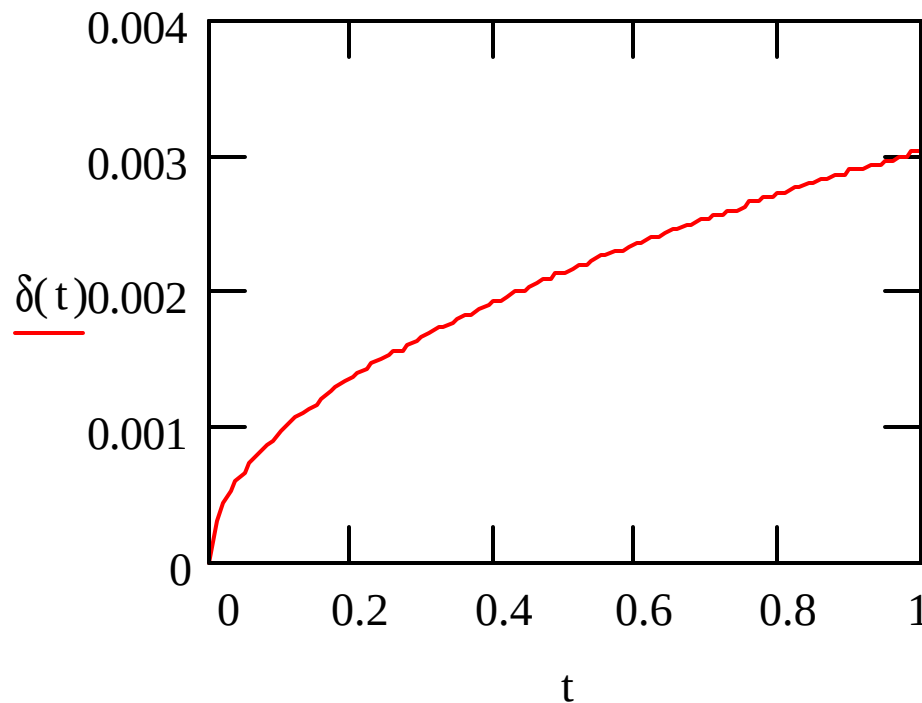
$$r h_{sf} \frac{dS}{dt} = q'' = \frac{k(T_f - T_i)}{\sqrt{p a t}}, \text{ and } \int_0^d dS = \frac{k(T_f - T_i)}{r h_{sf} \sqrt{p a}} \int_0^t \frac{dt}{\sqrt{t}}$$

$$d(t) = \frac{2k(T_f - T_i)}{r h_{sf} \sqrt{p a}} \sqrt{t}, \text{ therefore, } d \propto \sqrt{t}. \text{ Cooling time } t = \frac{p a}{4k^2} \left(\frac{d r h_{sf}}{T_f - T_i} \right)^2$$

Use the following values to calculate: $k=120 \text{ W/m.K}$, $\alpha=4 \times 10^{-5} \text{ m}^2/\text{s}$, $\rho=3970 \text{ kg/m}^3$, and $h_{sf}=3.577 \times 10^6 \text{ J/kg}$, $T_f=2318 \text{ K}$, $T_i=300\text{K}$, and $\delta=2 \text{ mm}$

Example (cont.)

$$d(t) = \frac{2k(T_f - T_i)}{rh_{sf}\sqrt{pa}}\sqrt{t} = 0.00304\sqrt{t}$$



- $\delta(t) \propto t^{1/2}$
- Therefore, the layer solidifies very fast initially and then slows down as shown in the figure
- Note: we neglect contact resistance between the coating and the substrate and assume temperature of the coating material stays the same even after it solidifies.

- To solidify 2 mm thickness, it takes 0.43 seconds.

Example (cont.)

What will be the substrate temperature as it varies in time? The temperature distribution is:

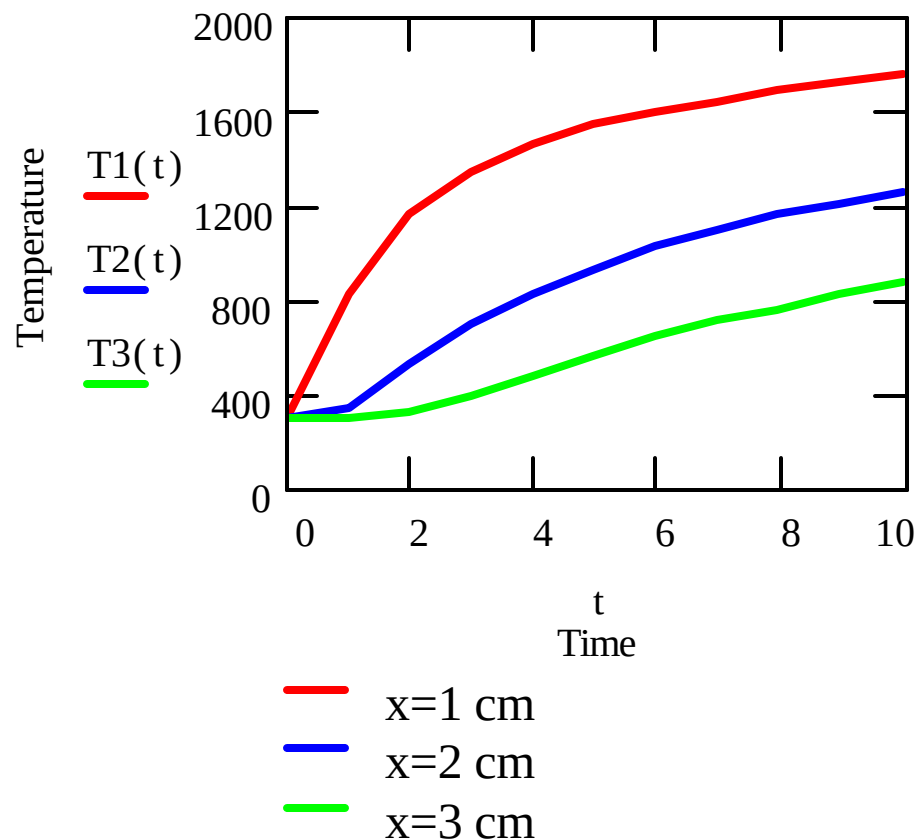
$$\frac{T(x,t) - T_s}{T_i - T_s} = \operatorname{erf}\left(\frac{x}{2\sqrt{at}}\right),$$

$$T(x,t) = 2318 + (300 - 2318)\operatorname{erf}\left(\frac{x}{2\sqrt{at}}\right) = 2318 - 2018\operatorname{erf}\left(79.06\frac{x}{\sqrt{t}}\right)$$

Example (cont.)

- For a fixed distance away from the surface, we can examine the variation of the temperature as a function of time. Example, 1 cm deep into the substrate the temperature should behave as:

$$T(x = 0.01, t) = 2318 - 2018 \operatorname{erf}\left(\frac{79.06}{\sqrt{t}}\right) = 2318 - 2018 \operatorname{erf}\left(\frac{0.79}{\sqrt{t}}\right)$$

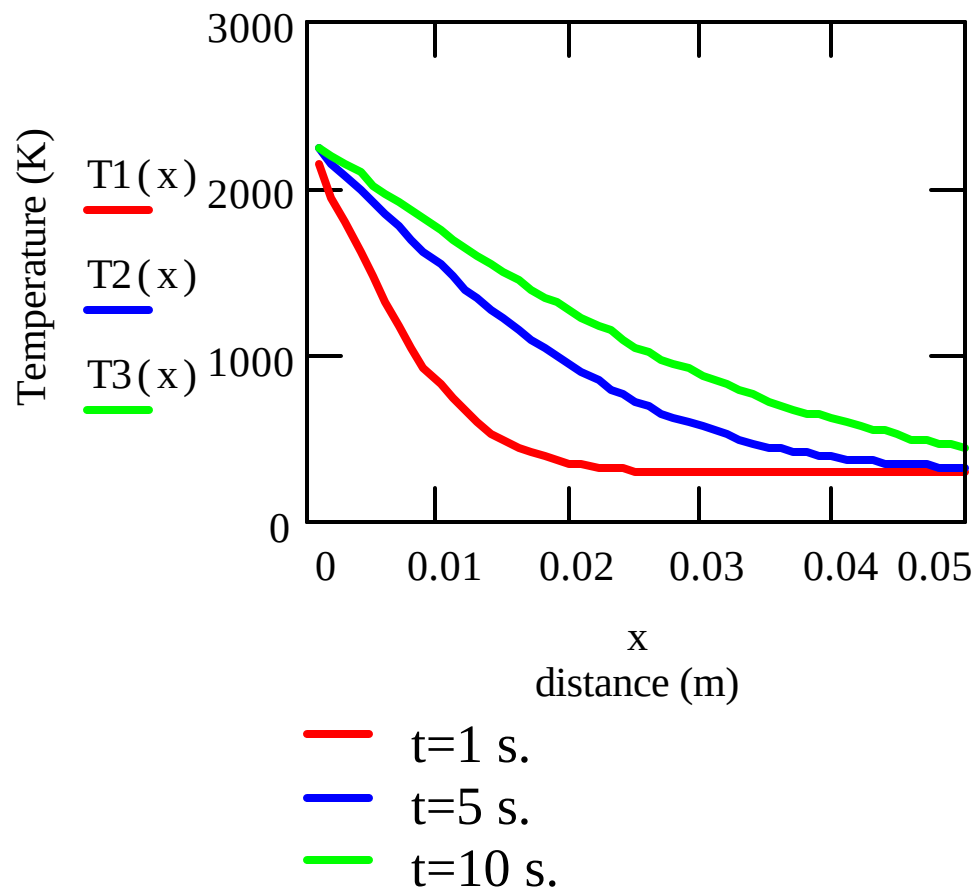


- At $x=1$ cm, the temperature rises almost instantaneously at a very fast rate. A short time later, the rate of temp. increase slows down significantly since the energy has to distribute to a very large mass.
- At deeper depth ($x=2$ & 3 cm), the temperature will not respond to the surface condition until much later.

Example (cont.)

We can also examine the spatial temperature distribution at any given time, say at $t=1$ second.

$$T(x, t = 1) = 2318 - 2018 \operatorname{erf}\left[79.06 \frac{x}{\sqrt{t}}\right] = 2318 - 2018 \operatorname{erf}(79.06x)$$



- Heat penetrates into the substrate as shown for different time instants.
- It takes more than 5 seconds for the energy to transfer to a depth of 5 cm into the substrate
- The slopes of the temperature profiles indicate the amount of conduction heat transfer at that instant.