

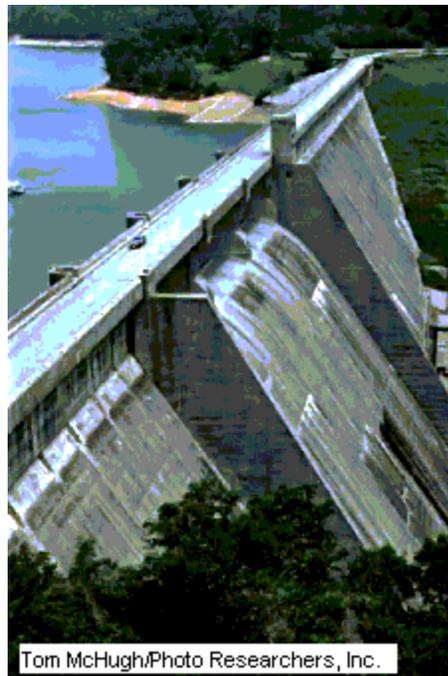
Arch Dam



Dams

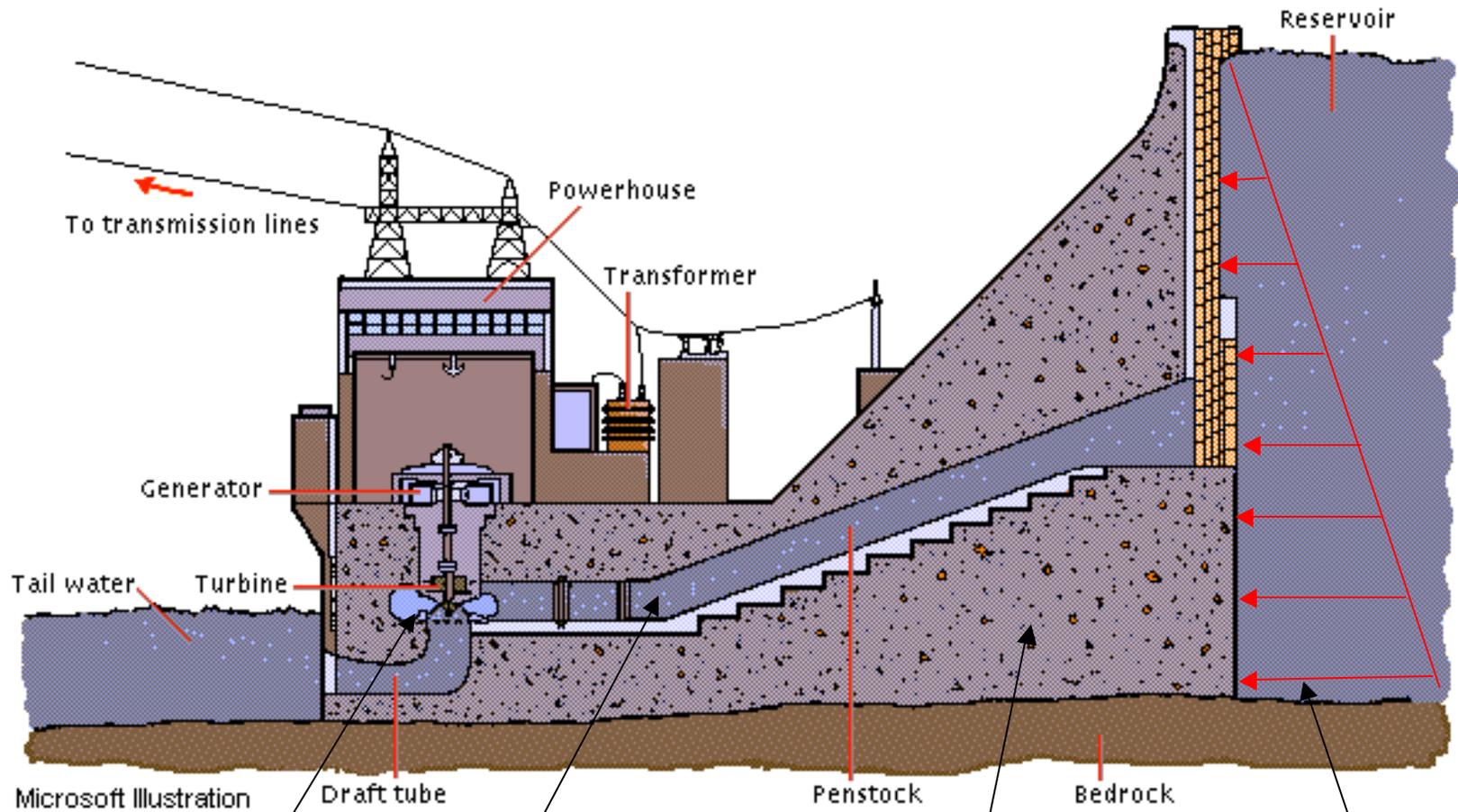


Arch & Gravity Dam



Gravity Dam

Dams (cont.)



Turbine

Energy conversion
Bernoulli equation

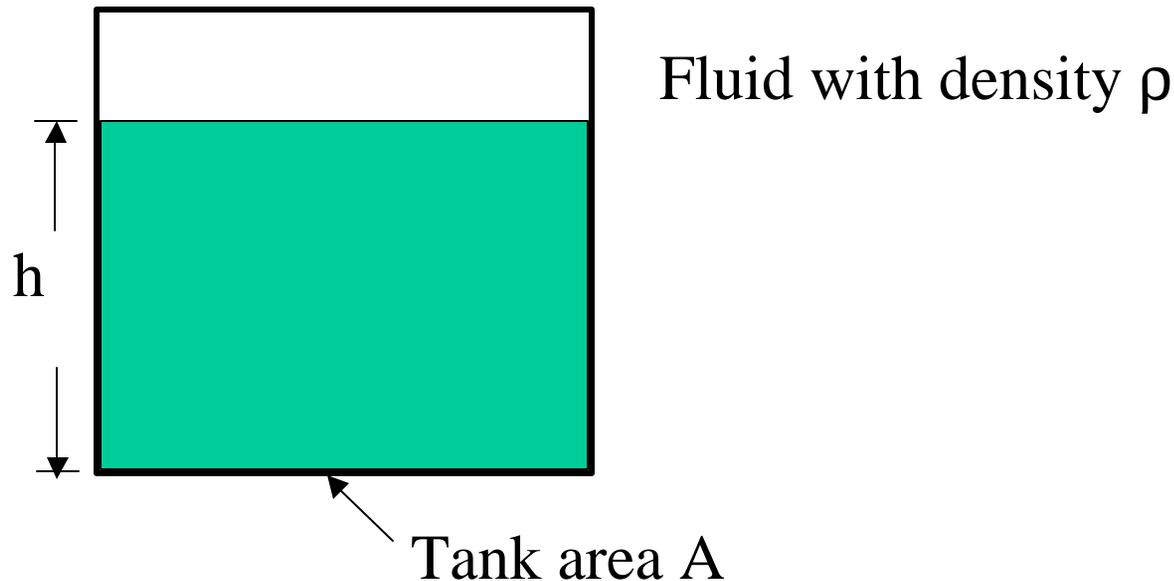
Hydrostatic uplift

Hydrostatic forces

Fluid Statics

When a surface is submerged in a fluid at rest, hydrostatic forces develop on the surface due to the fluid pressure. These forces must be perpendicular to the surface since there is no shear action present. These forces can be determined by integrating the static pressure distribution over the area it is acting on.

Example: What is the force acting on the bottom of the tank shown?



Dam Design

Design concern: (**Hydrostatic Uplift**) Hydrostatic pressure above the heel (upstream edge) of the dam may cause seepage with resultant uplift beneath the dam base (depends largely on the supporting material of the dam). This reduces the dam's stability to sliding and overturning by effectively reducing the weight of the dam structure. (Question: What prevents the dam from sliding?)

Determine the minimum compressive stresses in the base of a concrete gravity dam as given below. It is important that this value should be greater than zero because (1) concrete has poor tensile strength. Damage might occur near the heel of the dam. (2) The lifting of the dam structure will accelerate the seeping rate of the water underneath the dam and further increase hydrostatic uplift and generate more instability.

Catastrophic breakdown can occur if this factor is not considered: for example, it is partially responsible for the total collapse of the St. Francis Dam in California, 1928.

Dam Design

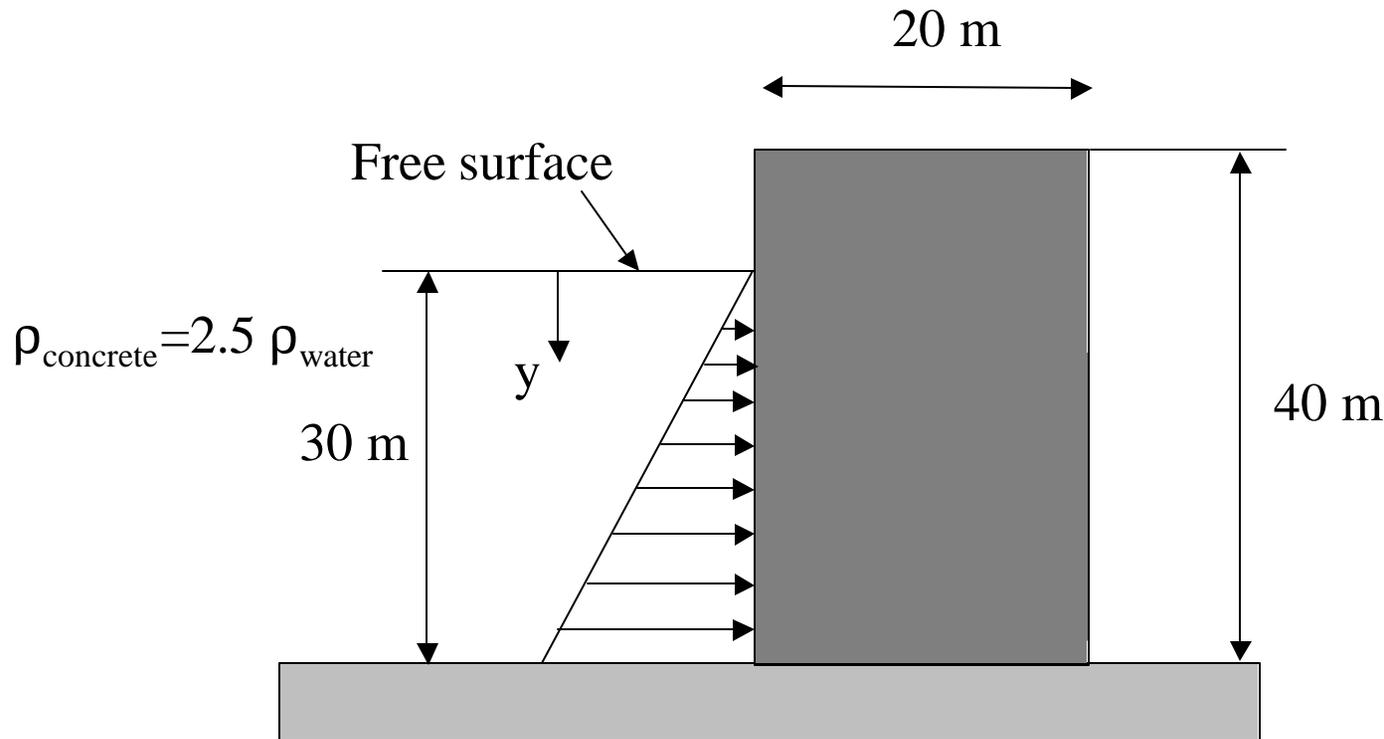
First, calculate the weight of the dam (per unit width):

$$W = \rho V g = (2.5)(1000)(20)(40)(1)(9.8) = 19.6 \times 10^6 \text{ (N)}$$

The static pressure at a depth of y : $P(y) = \rho_w g y$

The total resultant force acting on the dam by the water pressure is:

$$R = \int P(y) dy = \int_0^{h=30} \rho_w g y dy = \rho_w g \left(\frac{h^2}{2} \right) = (1000)(9.8)(1/2)(30)^2 = 4.4 \times 10^6 \text{ (N)}$$



Example (cont.)

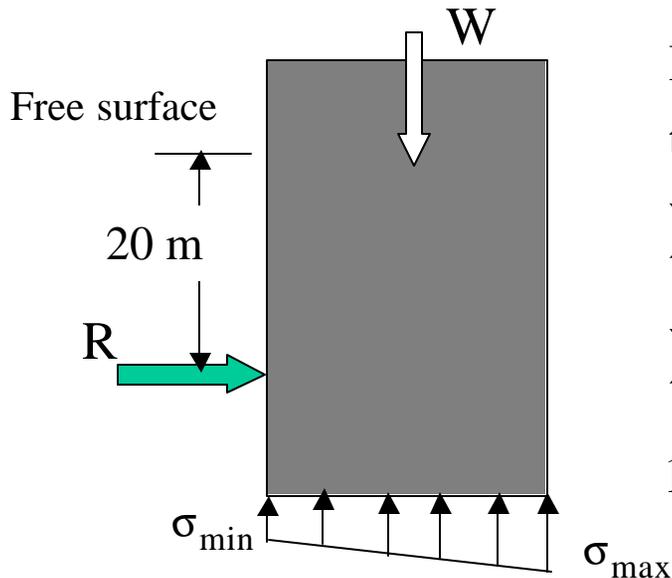
The resultant force, R , is acting at a depth \bar{h} below the free surface so that

$$R\bar{h} = \int P(y)ydy = \int_0^{h=30} (r_w g y) y dy = r_w g \int_0^{30} y^2 dy = r_w g \frac{h^3}{3}, \quad \bar{h} = \frac{r_w g \frac{h^3}{3}}{R} = \frac{2h}{3} = 20(m)$$

Assume the load distribution under the dam is linear (it might not be linear if the soil distribution is not uniform)

Therefore, the stress distribution can be written as

$$s(x) = s_{\min} + \frac{s_{\max} - s_{\min}}{20} x$$



In order to reach equilibrium, both the sum of forces and the sum of moments have to balance to zero

$$\sum F_x = 0, \quad R = F_{\text{dam},x} \text{ (frictional force and the air drag force)}$$

$$\sum F_y = 0, \quad W = F_{\text{dam},y} = \int_0^{20} s(x) dx = 10(s_{\max} + s_{\min})$$

$$1.96 \times 10^6 (N) = s_{\max} + s_{\min}$$

Example (cont.)

The sum of moments has to be zero also: Taking moment w.r.t. the heel of the dam

$$\sum M_o = 0, \quad -R(10) - W(10) + \int_0^{20} s(x)x dx = 0$$

$$(10)(4.4 \times 10^6 + 19.6 \times 10^6) = s_{\max} \int_0^{20} x dx + \frac{s_{\max} - s_{\min}}{20} \int_0^{20} x^2 dx$$

$$240 \times 10^6 = 133.3s_{\max} + 66.7s_{\min}$$

$$\text{Solve: } s_{\max} = 1.64 \times 10^6 (N), \quad s_{\min} = 0.32 \times 10^6 (N)$$

The minimum compressive stress is significantly lower than the maximum stress

The hydrostatic lift under the dam (as a result of the buoyancy induced by water seeping under the dam structure) can induce as high as one half of the maximum hydrostatic head at the heel of the dam and gradually decrease to zero at the other end.

$$\text{That is } s_{\text{lift}} = \frac{1}{2}(r_w gh) = (0.5)(1000)(9.8)(30) = 0.147 \times 10^6 (N)$$

Therefore, the effective compressive stress will only be $0.173(=0.32-0.147) \times 10^6 (N)$.