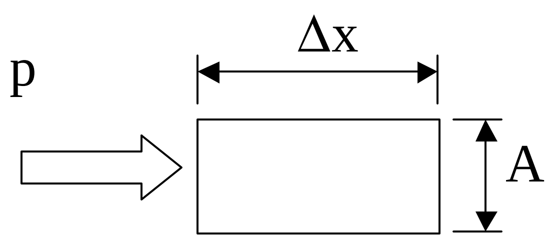
Energy Conservation (Bernoulli's Equation)

Integration of Euler's equation $\int_1^2 \frac{dp}{r} + \int_1^2 VdV + \int_1^2 gdz = 0$

Bernoulli's equation $\boxed{\frac{p_1}{r}} + \frac{V_1^{\,2}}{2} + gz_1 = \frac{p_2}{r} + \frac{V_2^{\,2}}{2} + gz_2$

$\boxed{\text{Flow work}}$ + kinetic energy + potential energy = constant



Under the action of the pressure, the fluid element moves a distance Δx within time Δt
The work done per unit time ΔW/Δt (flow power) is

$$\frac{\Delta W}{\Delta t} = \frac{pA\Delta x}{\Delta t} = \left(\frac{p}{r}\right) rA\frac{\Delta x}{\Delta t} = rAV\left(\frac{P}{r}\right),$$

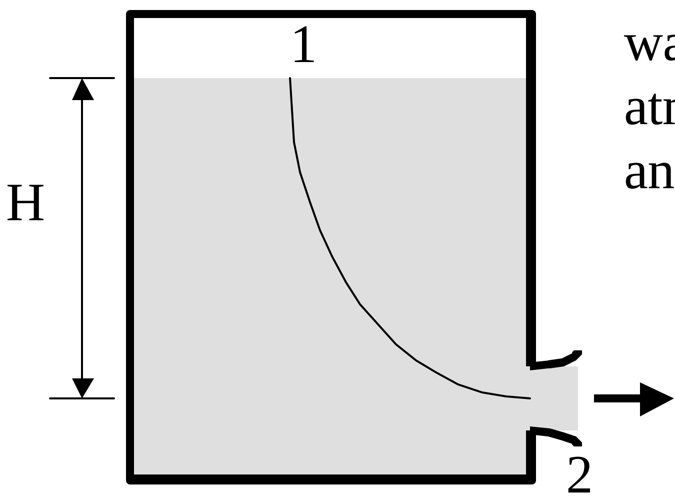
$$\frac{p}{r} = \left(\frac{1}{rAV}\right)\left(\frac{\Delta W}{\Delta t}\right) = \text{work done per unit mass flow rate}$$

Energy Conservation (cont.)

$$\frac{p_1}{g}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{g}+\frac{V_2^2}{2g}+z_2, \text{ where } g = r\text{g (energy per unit weight)}$$

It is valid for incompressible fluids, steady flow along a streamline, no energy loss due to friction, no heat transfer.

Examples:



Determine the velocity and mass flow rate of efflux from the circular hole (0.1 m dia.) at the bottom of the water tank (at this instant). The tank is open to the atmosphere and H=4 m

$p_1 = p_2$, $V_1=0$

$$V_2=\sqrt{2g(z_1-z_2)}=\sqrt{2gH}$$

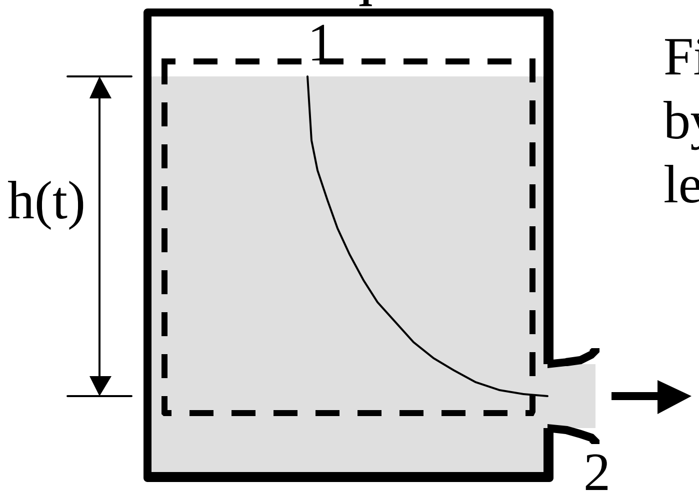
$$=\sqrt{2*9.8*4}=8.85(m/s)$$

$$\dot{m}=rAV=1000*\frac{p}{4}(0.1)^2(8.85)$$

$$=69.5(kg/s)$$

Energy Equation(cont.)

Example: If the tank has a cross-sectional area of 1 m^2, estimate the time required to drain the tank to level 2.



First, choose the control volume as enclosed by the dotted line. Specify h=h(t) as the water level as a function of time.

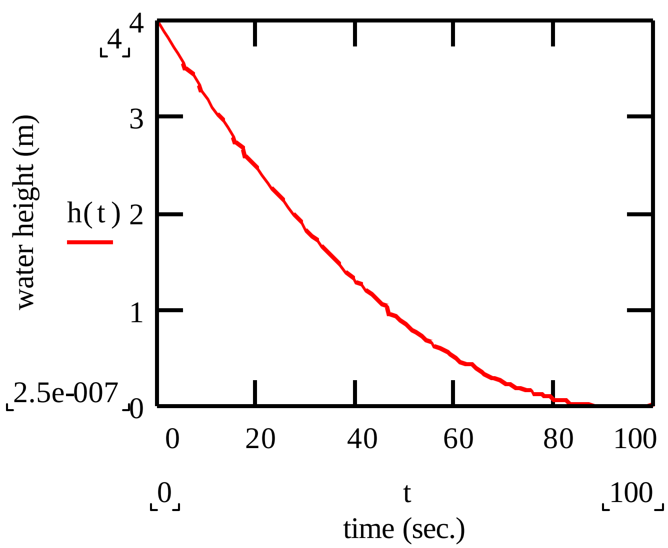
From Bernoulli' s equation, $V = \sqrt{2gh}$

From mass conservation, $\dfrac{\mathrm{dm}}{\mathrm{dt}} = -rA_{hole}V$

since $m = rA_{\tan k}h$, $\dfrac{dh}{dt} = -\dfrac{A_{hole}}{A_{\tan k}}V = -\dfrac{(0.1)^2}{1^2}\sqrt{2gh}$

$\dfrac{dh}{dt} = -0.0443\sqrt{h}$, $\dfrac{\mathrm{dh}}{\sqrt{\mathrm{h}}} = -0.0443dt$, integrate

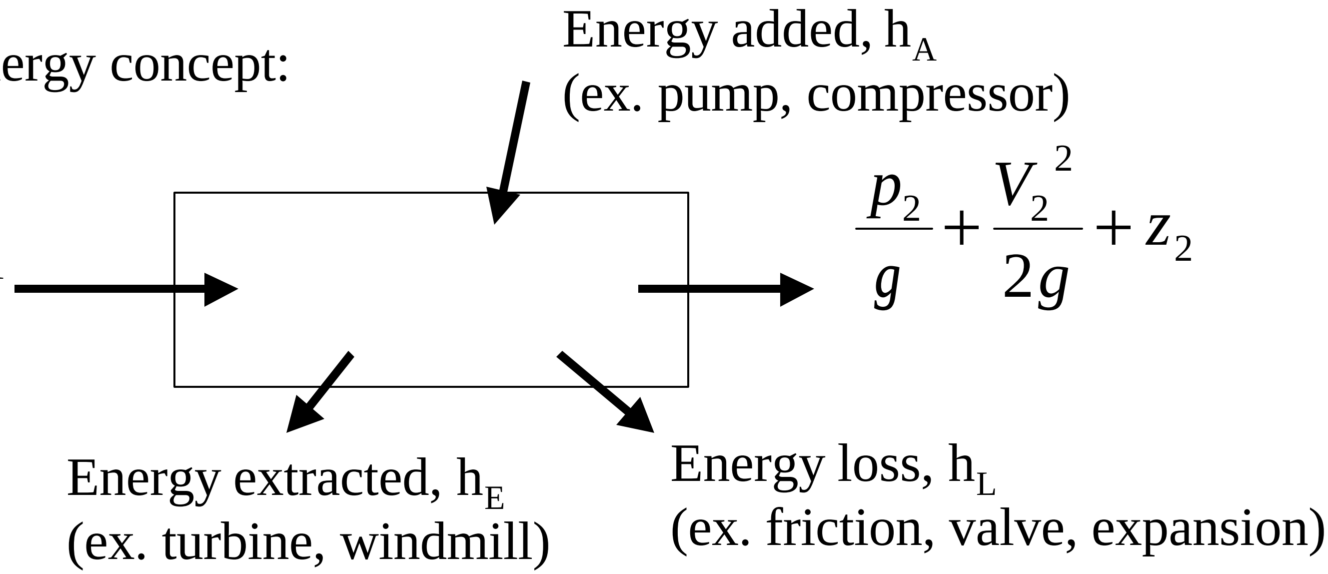
$\sqrt{\mathrm{h(t)}} = \sqrt{\mathrm{H}} - 0.0215t$, $h = 0$, $t_{drain} = 93$ sec.



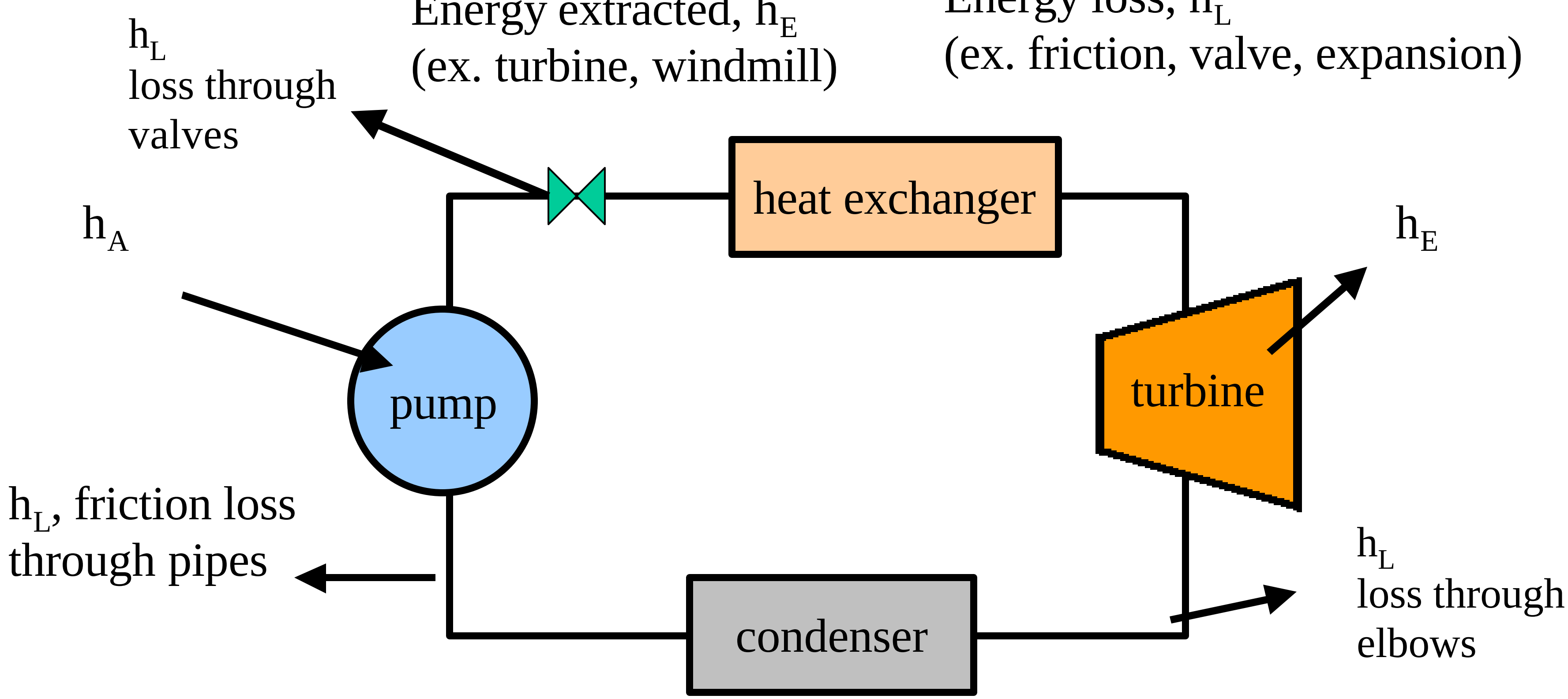
Energy conservation (cont.)

Generalized energy concept:

$$\frac{p_1}{g}+\frac{V_1^{\,2}}{2g}+z_1$$



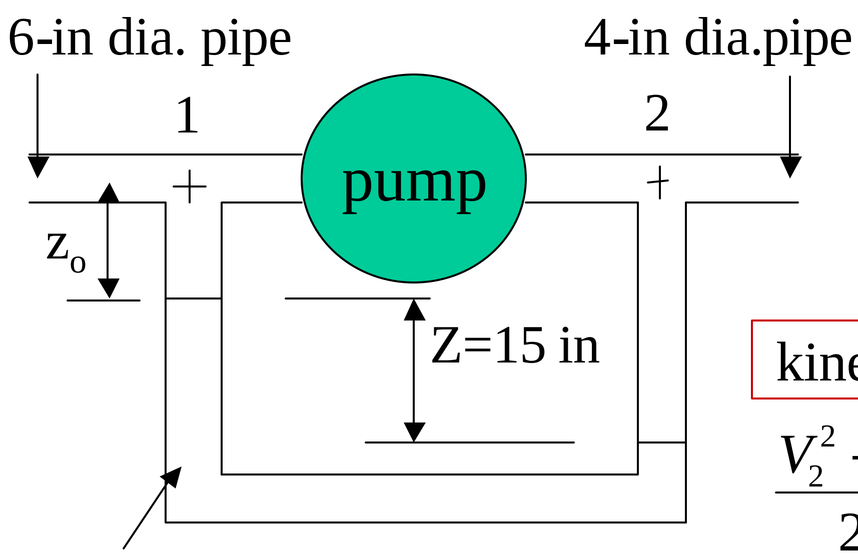
$$\frac{p_2}{g}+\frac{V_2^{\,2}}{2g}+z_2$$



Energy conservation(cont.)

Extended Bernoulli's equation, $\frac{p_1}{g}+\frac{V_1^2}{2g}+z_1+h_A-h_E-h_L=\frac{p_2}{g}+\frac{V_2^2}{2g}+z_2$

Examples: Determine the efficiency of the pump if the power input of the motor is measured to be 1.5 hp. It is known that the pump delivers 300 gal/min of water.



Mercury (γ_m=844.9 lb/ft^3)
water (γ_w=62.4 lb/ft^3)
1 hp=550 lb-ft/s

$\boxed{h_E=h_L=0,\ \ z_1=z_2}$

Q=300 gal/min=0.667 ft^3/s=AV
V_1=Q/A_1=3.33 ft/s
V_2=Q/A_2=7.54 ft/s

kinetic energy head gain

$$\frac{V_2^2-V_1^2}{2g}=\frac{(7.54)^2-(3.33)^2}{2*32.2}=0.71ft,$$

$$p_1+g_wz_o+g_mz=p_2+g_wz_o+g_wz$$

$$p_2-p_1=(g_m-g_w)z$$

$$=(844.9-62.4)*1.25=978.13\,lb/\ ft^2$$

Energy conservation (cont.)

Example (cont.)

Pressure head gain:

$$\frac{p_2 - p_1}{g_w} = \frac{978.13}{62.4} = 15.67\,(ft)$$

$$\text{pump work } h_A = \frac{p_2 - p_1}{g_w} + \frac{V_2^2 - V_1^2}{2g} = 16.38(ft)$$

Flow power delivered by pump

$$\mathrm{P} = g_{\mathrm{w}} Q h_A = (62.4)(0.667)(16.38)$$

$$= 681.7(ft - lb/s)$$

$$1hp = 550\,ft - lb/s$$

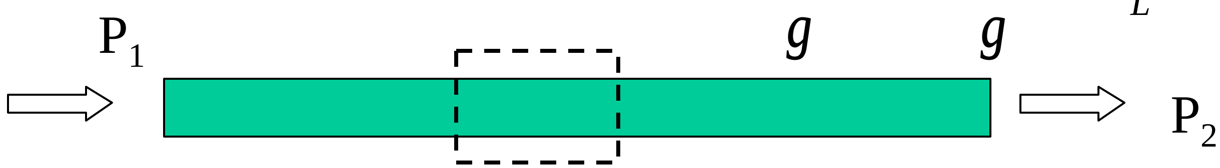
$$P = 1.24hp$$

$$\text{Efficiency } h = \frac{\mathrm{P}}{\mathrm{P}_{\mathrm{input}}} = \frac{1.24}{1.5} = 0.827 = 82.7\%$$

Frictional losses in piping system

Extended Bernoulli's equation, $\dfrac{p_1}{g}+\dfrac{V_1^2}{2g}+z_1+h_A-h_E-h_L=\dfrac{p_2}{g}+\dfrac{V_2^2}{2g}+z_2$

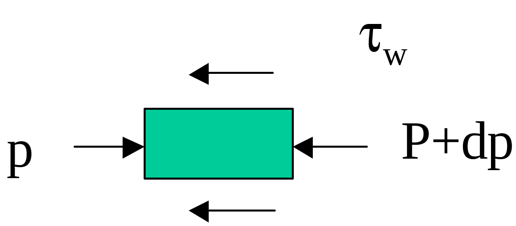
$$\frac{p_1-p_2}{g}=\frac{\Delta p}{g}=h_L=\text{frictional head loss}$$



Consider a laminar, fully developed circular pipe flow

R: radius, D: diameter
L: pipe length
τ_w: wall shear stress

$$[p-(p+dp)](\pi R^2)=\tau_w(2\pi R)dx,$$



Pressure force balances frictional force

$$-dp=\frac{2\tau_w}{R}dx,\ \text{integrate from 1 to 2}$$

Darcy's Equation: $\dfrac{\Delta p}{g}=\dfrac{p_1-p_2}{g}=h_L=\dfrac{4\tau_w}{\rho g}\left(\dfrac{L}{D}\right)=f\left(\dfrac{L}{D}\right)\left(\dfrac{V^2}{2g}\right)$

$$\tau_w=\left(\frac{f}{4}\right)\left(\frac{\rho V^2}{2}\right)$$

where f is defined as frictional factor characterizing pressure loss due to pipe wall shear stress

When the pipe flow is laminar, it can be shown (not here) that

$f = \frac{64\mu}{VD\rho}$, by recognizing that $\mathrm{Re} = \frac{\rho VD}{\mu}$, as Reynolds number

Therefore, $f = \frac{64}{\mathrm{Re}}$, frictional factor is a function of the Reynolds number

Similarly, for a turbulent flow, f = function of Reynolds number also

$f = F(\mathrm{Re})$. Another parameter that influences the friction is the surface

roughness as relativeto the pipe diameter $\frac{e}{\mathrm{D}}$.

Such that $f = F\left(\mathrm{Re}, \frac{e}{\mathrm{D}}\right)$: Pipe frictional factor is a function of pipe Reynolds

number and the relative roughness of pipe.

This relation is sketched in the Moody diagram as shown in the following page.

The diagram shows f as a function of the Reynolds number (Re), with a series of

parametric curves related to the relative roughness $\left(\frac{e}{\mathrm{D}}\right)$.

Energy Conservation (cont.)

Energy: E=U(internal thermal energy)+E_{mech} (mechanical energy)
=U+KE(kinetic energy)+PE(potential energy)
Work: W=W_{ext}(external work)+W_{flow}(flow work)
Heat: Q heat transfer via conduction, convection & radiation

dE=dQ-dW, dQ>0 net heat transfer in dE>0 energy increase and vice versa
dW>0, does positive work at the expense of decreasing energy, dE<0

U=mu, u(internal energy per unit mass), KE=(1/2)mV^2, PE=mgz
W_{flow}=m(p/ρ)

Energy flow rate: $\dot{m}(u+\frac{V^2}{2}+gz)$ plus Flow work rate $\dot{m}\left(\frac{p}{\rho}\right)$

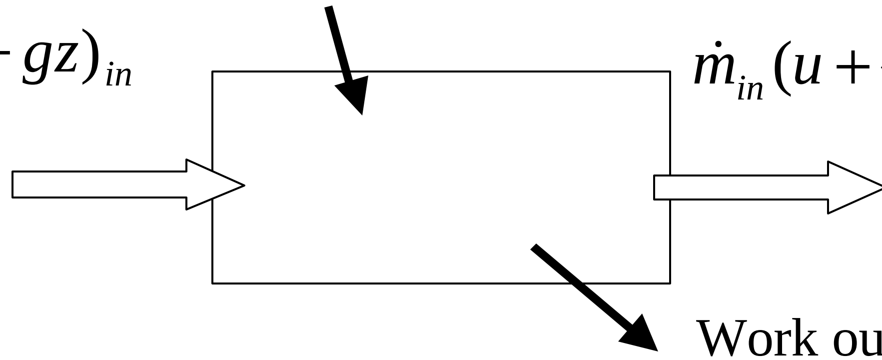
Flow energy in = $\dot{m}_{in}(u+\frac{p}{\rho}+\frac{V^2}{2}+gz)_{in}$, Energy out = $\dot{m}_{out}(u+\frac{p}{\rho}+\frac{V^2}{2}+gz)_{out}$

Their difference is due to external heat transfer and work done on flow

Energy Conservation (cont.)

Heat in q=dQ/dt

$$\dot{m}_{in}(u+\frac{p}{r}+\frac{V^2}{2}+gz)_{in}$$



$$\dot{m}_{in}(u+\frac{p}{r}+\frac{V^2}{2}+gz)_{out}$$

Work out dW/dt

From mass conservation: $\dot{m}_{in}=\dot{m}_{out}=\dot{m}$

From the First law of Thermodynamics (Energy Conservation):

$$\frac{\mathrm{dQ}}{\mathrm{dt}}+\dot{m}(u+\frac{p}{r}+\frac{V^2}{2}+gz)_{in}=\dot{m}(u+\frac{p}{r}+\frac{V^2}{2}+gz)_{out}+\frac{dW}{dt},\ \text{or}$$

$$\frac{\mathrm{dQ}}{\mathrm{dt}}+\dot{m}(h+\frac{V^2}{2}+gz)_{in}=\dot{m}(h+\frac{V^2}{2}+gz)_{out}+\frac{dW}{dt}$$

where $h=u+\frac{p}{r}$ is defined as "enthaply"

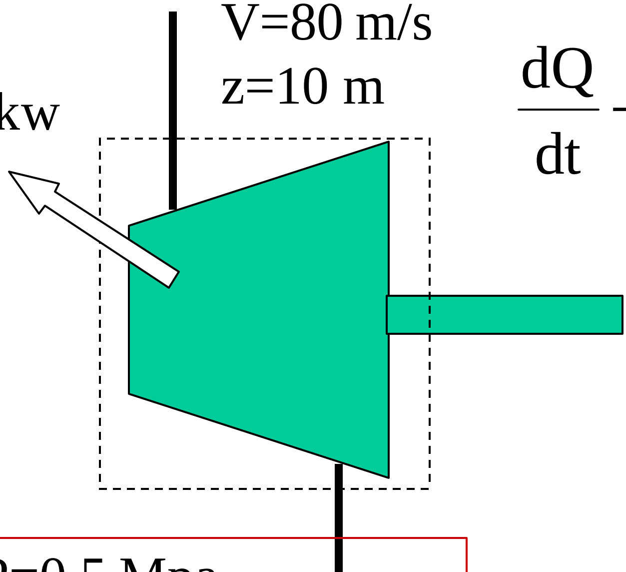
Energy Conservation(cont.)

Example: Superheated water vapor is entering the steam turbine with a mass flow rate of 1 kg/s and exhausting as saturated steamas shown. Heat loss from the turbine is 10 kW under the following operating condition. Determine the power output of the turbine.

P=1.4 Mpa
T=350° C
V=80 m/s
z=10 m

From superheated vapor table:
h_{in}=3149.5 kJ/kg

10 kw



P=0.5 Mpa
100% saturated steam
V=50 m/s
z=5 m

$$\frac{\mathrm{dQ}}{\mathrm{dt}} + \dot{m}\left(h + \frac{V^2}{2} + gz\right)_{in} = \dot{m}\left(h + \frac{V^2}{2} + gz\right)_{out} + \frac{dW}{dt}$$

$$\frac{dW}{dt} = (-10) + (1)[(3149.5 - 2748.7) + \frac{80^2 - 50^2}{2(1000)} + \frac{(9.8)(10-5)}{1000}]$$

$$= -10 + 400.8 + 1.95 + 0.049$$

$$= 392.8(kW)$$

From saturated steam table: h_{out}=2748.7 kJ/kg