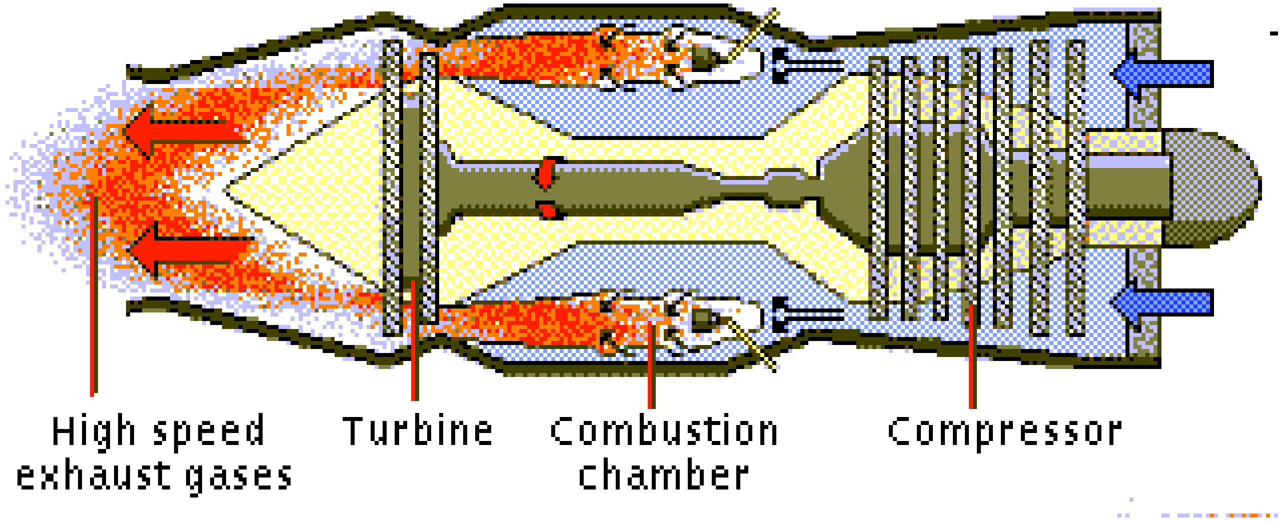
General Formulation - A Turbojet Engine



Relevant physical quantities: mechanical forces such as thrust, drag, lift; thermodynamic properties such as energy, enthalpy, entropy in order to calculate thermal loading and efficiency; mass flow rate and mass/fuel ratio, temperature and pressure distributions for the estimation of heat transfer and engine performance.

Governing Equations

Conservation Principles:

Mass conservation: mass is conserved. (continuity equation)

➔ 1 scalar equation

Momentum conservation: Newton’s second law: $\sum F=ma$.

➔ 1 vector equation, or 3 scalar equations, one in each direction

Energy conservation: Second law of Thermodynamics.

➔ 1 scalar equation.

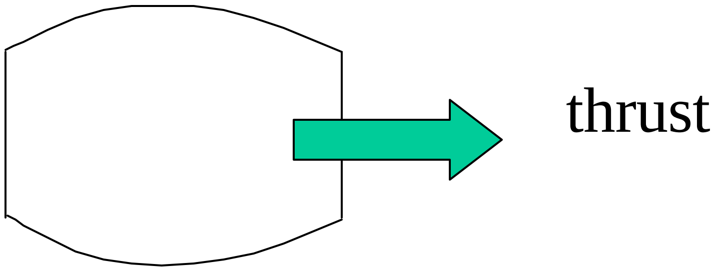
Equation of state:

Relate pressure to the density and the temperature: example: the ideal gas law, $P=\rho RT$ ➔ 1 scalar equation.

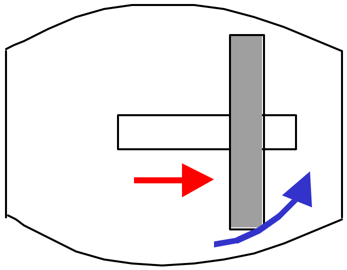
Fluid properties to be analyzed: pressure (P), velocity (three components U, V, W), density(ρ), temperature (T).

Six partial differential equations for six unknowns, can be solved if proper boundary and initial conditions are given

Integral and Differential Forms



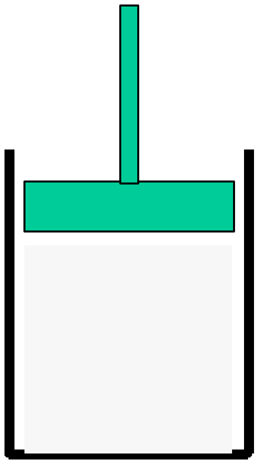
Estimate of the **integral** effects acting on the system by analyzing the interaction between the fluids and the flow devices. Ex: thrust produced by the jet engine.



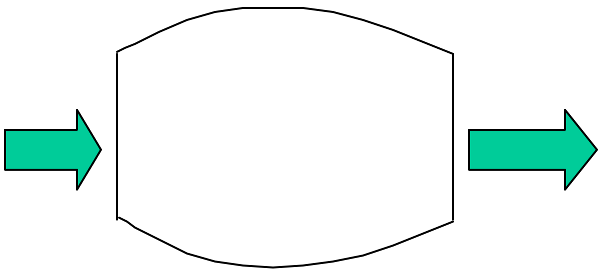
Forces (normal and shear stresses) and flow patterns over individual blades.

Governing equations in **differential form** are needed to analyze the flow pattern and its local interaction with the solid surface.

System and Control Volume



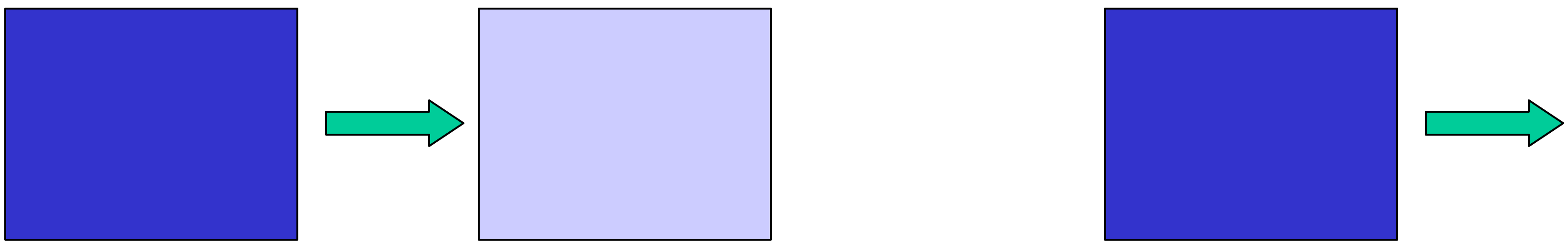
A system must always contain the same matter. Example: A piston between intake and exhaust strokes. Analysis of fluid properties based on the system is called Lagrangian approach.



A control volume: A definite volume specified in space. Matter in a control volume can change with time as matter enters and leaves its control surface. Example: A jet engine. This type of analysis is called Eulerian approach. Preferred in fluid mechanics.

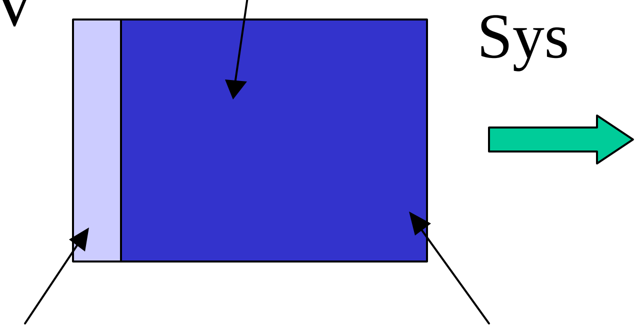
Reynolds Transport Theorem (system ➔ control volume)

System (Sys) Control Volume (CV) Sys+CV



Earlier time Present time (t)

Later time (t+Δt)



Mass conservation: $\left.\frac{\mathrm{dM}}{\mathrm{dt}}\right)_{Sys} = 0$

$$M = \int r \, d\forall$$

$$\mathrm{M}_{Sys,t+\Delta t} = (M_{II} + M_{III})_{t+\Delta t}$$

$$= [(M_{CV} - M_{I}) + M_{III}]_{t+\Delta t}$$

Also, $\mathrm{M}_{\mathrm{CV,\,t}} = M_{Sys,t}$

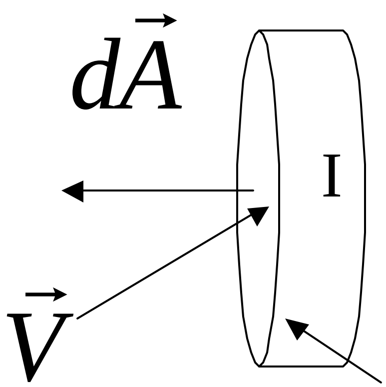
Mass Conservation

$$\left.\frac{dM}{dt}\right)_{Sys} = \lim_{\Delta t\to 0}\left.\frac{M_{t+\Delta t}-M_t}{\Delta t}\right)_{sys} = \lim_{\Delta t\to 0}\frac{[M_{CV}-M_I+M_{III}]_{t+\Delta t}-M_{Sys,t}}{\Delta t}$$

$$= \lim_{\Delta t\to 0}\frac{\left.M_{CV}\right)_{t+\Delta t}-\left.M_{CV}\right)_t}{\Delta t} + \lim_{\Delta t\to 0}\frac{\left.M_{III}\right)_{t+\Delta t}}{\Delta t} - \lim_{\Delta t\to 0}\frac{\left.M_I\right)_{t+\Delta t}}{\Delta t}$$

$$= \frac{\partial}{\partial t}M_{CV} + \lim_{\Delta t\to 0}\frac{\left.M_{III}\right)_{t+\Delta t}}{\Delta t} - \lim_{\Delta t\to 0}\frac{\left.M_I\right)_{t+\Delta t}}{\Delta t}$$

$$= \frac{\partial}{\partial t}\int_{CV} r\, d\forall + \text{ mass out - mass in}$$

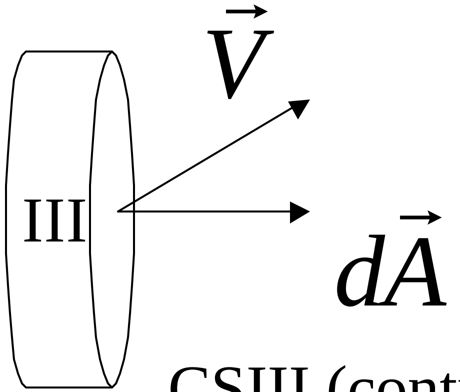


Mass in = $-\int_{CSI} r\,\vec{V}\cdot d\vec{A}$: minus sign because

the area sign $d\vec{A}$ is positive pointing outwards

CSI (control surface)

Mass Conservation



CSIII (control surface)

Similarly, Mass out = $\int_{CSIII} r\vec{V}\cdot d\vec{A}$

plus sign in the same direction as $d\vec{A}$

$$\left.\frac{dM}{dt}\right)_{Sys} = \frac{\partial}{\partial t}\int_{CV} r\,d\forall + \text{ mass out - mass in}$$

$$= \frac{\partial}{\partial t}\int_{CV} r\,d\forall + \int_{CSIII} r\vec{V}\cdot d\vec{A} + \int_{CSI} r\vec{V}\cdot d\vec{A} = \frac{\partial}{\partial t}\int_{CV} r\,d\forall + \int_{CS} r\vec{V}\cdot d\vec{A}$$

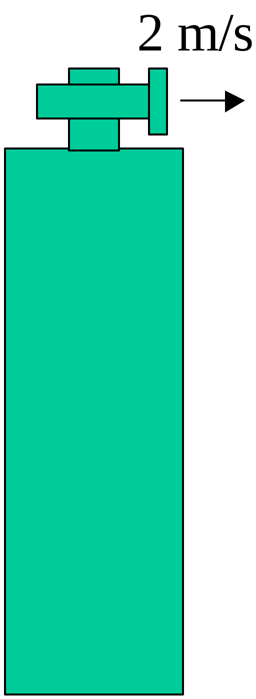
From mass conservation: $\left.\frac{dM}{dt}\right)_{Sys} = 0$

Therefore, $\frac{\partial}{\partial t}\int_{CV} r\,d\forall + \int_{CS} r\vec{V}\cdot d\vec{A} = 0$

Mass Conservation in control volume form

Example

Metal can be cut by using an oxyacetylene torch. During the process, oxygen is supplied via a pressurized tank (30 cm in dia. And 1 m tall). A valve is used to maintain the oxygen flowing out at a constant velocity of 2 m/s. The valve has an opening area of 10^{-4} m^2 and the temperature inside the tank is unchanged and equal to 25°C. If the initial pressure inside the tank is 10,000 kPa gage, how long will it take to drain the tank such that the pressure is 90% of its initial value.



Select the control volume as the tank and the valve opening as the only control surface that oxygen escapes.

Mass conservation: $\frac{\partial}{\partial t}\int_{CV} r\, d\forall + \int_{CS} r\vec{V}\cdot d\vec{A} = 0$

Since the density is uniform inside the tank: $m_{CV} = r\forall$

Also, assume oxygen is an idea gas: $r = \frac{P}{RT}$

If the temperature is a constant:

$$\frac{\partial}{\partial t}\int_{CV} r\, d\forall = \frac{\partial}{\partial t}(r\forall) = \frac{\forall}{RT}\frac{dP}{dT}.$$

Example (cont.)

$$\frac{\partial}{\partial t}\int_{CV} r\, d\forall = \frac{\forall}{RT}\frac{dP}{dt} = 9.1\times 10^{-7}\frac{dP}{dt},\quad \text{R=260 (J/kg.K) for oxygen}$$

The mass flow leaving the valve is $\int_{\mathrm{CS}} r\,\vec{\mathrm{V}}\cdot \mathrm{d}\vec{\mathrm{A}} = -r\,A_{valve}V = -\frac{P}{RT}(10^{-4})(2)$

$= -2.58\times 10^{-9}P$

$$9.1\times 10^{-7}\frac{dP}{dt} = -2.58\times 10^{-9}P,\quad \frac{dP}{P} = -0.00284dt$$

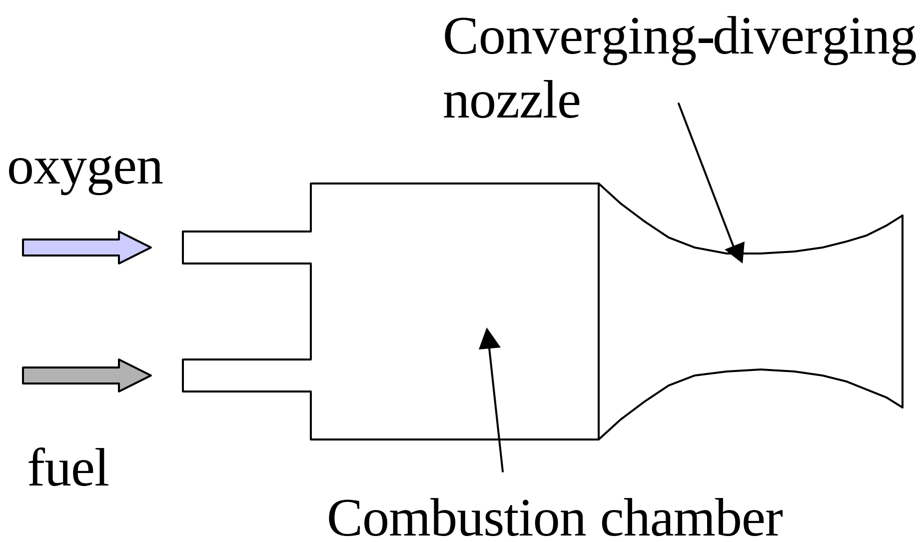
Integrate $\int_{P=1000}^{9000}\frac{dP}{P} = \ln\left(\frac{9000}{1000}\right) = -0.105 = -0.00284t$

$t = 36.97(\text{sec.})$

It will take approximately 37 sec. to drain 10% of the oxygen from the tank

More Example

In a rocket engine, liquid fuel and liquid oxygen are fed into the combustion chamber as shown below. The hot exhaust gases of the combustion flow out of the nozzle at high exit velocity of 1000 m/s. Assume the pressure and temperature of the gases at the nozzle exit are 100 kPa and 800 K, respectively. The nozzle has an exit area of 0.05 m^2. Determine the mass flow rate of the fuel if the oxygen and fuel ratio is set at 5 to 1. Also, assume the exhaust gases can be modeled as an ideal gas with R=1 kJ/kg.K.



Shuttle Launch

Example (cont.)

Mass conservation: $\frac{\partial}{\partial t}\int_{CV} r\, d\forall + \int_{\mathrm{CS}} r\,\vec{\mathrm{V}}\cdot \mathrm{d}\vec{\mathrm{A}} = 0$

Assume steady state inside the rocket: $\int_{\mathrm{CS}} r\,\vec{\mathrm{V}}\cdot \mathrm{d}\vec{\mathrm{A}} = 0$

$$\int_{\mathrm{exit}} r\,\vec{\mathrm{V}}\cdot \mathrm{d}\vec{\mathrm{A}} = \dot{m}_{oxygen} + \dot{m}_{fuel} = (5+1)\dot{m}_{fuel}$$

$$r_e V_e A_e = \frac{P_e}{RT_e} V_e A_e = \frac{100000}{(1000)(800)}(1000)(0.05) = 6\dot{m}_{fuel}$$

$$\dot{m}_{fuel} = 6.25(kg/s)$$

$$\dot{m}_{oxygen} = 31.25(kg/s)$$