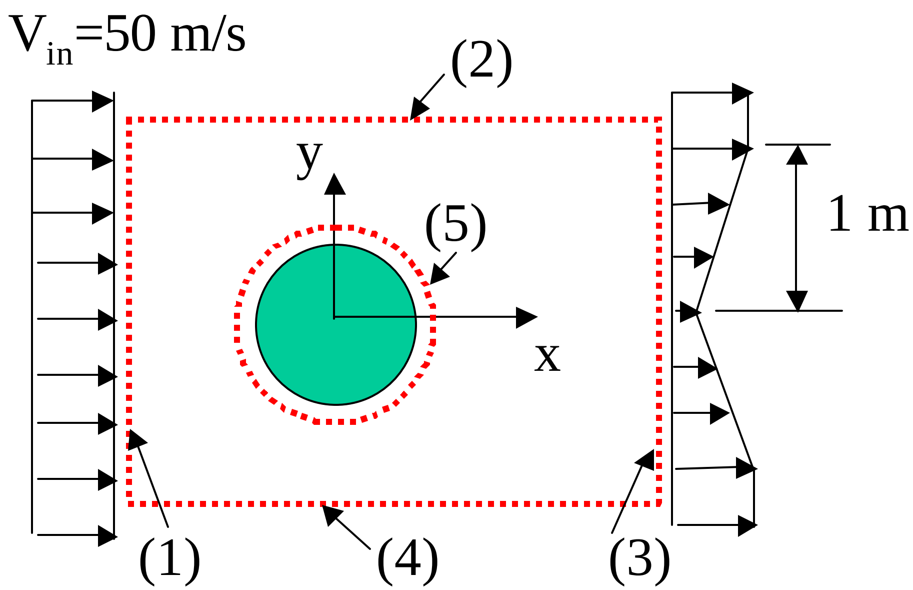
Drag and Momentum Balance

Aerodynamic drag is exerted on an object when fluid flow passes through it. This force is due to a combination of the shear and pressure forces acting on the surface of the object. The determination of these forces is difficult since it involves the measurement of both velocity and pressure fields near the surface of the object. However, based on the momentum balance concept, this force can also be determined as carrying out the momentum balance around the object. Drag force on a circular cylinder (unit width 1m) will be given here as an example. As shown below, velocity profiles before and after the cylinder are measured. Determine the drag force acting on the cylinder by assuming uniform pressure at the measuring stations.



$u(y) = 50$ (m/s), $|y| > 1$

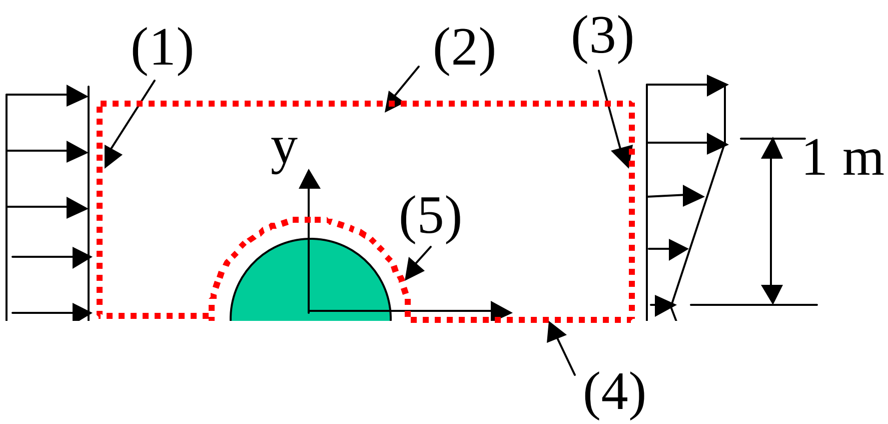
$= 20 + 30|y|$ (m/s), $|y| \le 1$

Control Volume and Control Surfaces

Consider a control volume surrounding the cylinder as shown in the previous slide. Assume the flow around the cylinder is symmetric, therefore, only half of the control volume is needed as shown below. There are a total of five control surfaces in this problem.

• Surface 4 is the symmetric plane, therefore, does not contribute since no flow (therefore momentum) going through it.

• Surface 5 is the surface surrounding the cylinder, and the integration of the pressure and shear stresses on 5 will give the total force the cylinder is acting on the fluid: R_x. This force should balance with the forces acting on surfaces (1), (2) & (3) plus the momentum flow in & out of those surfaces. (Note: the force in y direction will be zero, why?)



• The surface forces on surfaces (1) & (3) will be free-stream pressure and they should cancel.

• Momentum flow in & out of (1) & (3) can be determined by integration.

• Question: Is there momentum flow out of surface (2)?

Mass Conservation

Obviously, since there is more mass flows into (1) than that flows out of (3). Their difference is the mass flow out of surface (2). If there is mass flow then the momentum flow is nozero. Use mass conservation:
mass flow in (1) = mass flow out (2) + mass flow out (3)

$$\int_{CS} r\ \vec{V}\cdot d\vec{A} = 0, \Rightarrow \int_{(1)} r\ \vec{V}\cdot d\vec{A} + \int_{(2)} r\ \vec{V}\cdot d\vec{A} + \int_{(3)} r\ \vec{V}\cdot d\vec{A} = 0,$$

$$\text{or } \dot{m}_1 + \dot{m}_2 + \dot{m}_3 = 0$$

$$-rV_1A_1 + \dot{m}_2 + \int_0^{y=H} r\,u\,dA = 0, \quad \dot{m}_2 = r[V_1A_1 - \int_0^1 (20+30y)(1)dy - \int_1^H V_1 dA]$$

$$\dot{m}_2 = r[V_1(1)(H) - (20y+15y^2)\big|_0^1 - V_1(1)(H-1)]$$

$$= (1.2)[(50)(1) - (35)] = 18(kg/s)$$

Mass flow leaving the upper surface (2) is 18 (kg/s) and equal amount of mass is expected to leave the lower half of the control surface.

Momentum Conservation

The force acting on the cylinder can be evaluated

using the linear momentum balance:

$$\vec{F}_S + \vec{F}_B = \frac{\partial}{\partial t}\int_{CV} r\vec{V}d\forall + \int_{CS} r\vec{V}(\vec{V}\cdot d\vec{A}) = \int_{(1)}(\quad) + \int_{(2)}(\quad) + \int_{(3)}(\quad) + \int_{(4)}(\quad) + \int_{(5)}(\quad)$$

Only need to evaluate the force in the x direction:

$$\int_{(5)}(P+s\;)_x dA + \int_{(1)} P_x dA - \int_{(3)} P_x dA = \int_{(1)}(\quad) + \int_{(2)}(\quad) + \int_{(3)}(\quad)$$

$$R_x + P_{atm}A_1 - P_{atm}A_3 = -\int_{(1)} rV_1^2 dA + \int_{(2)} r\vec{V}(\vec{V}\cdot d\vec{A}) + \int_{(3)} rV^2 dA$$

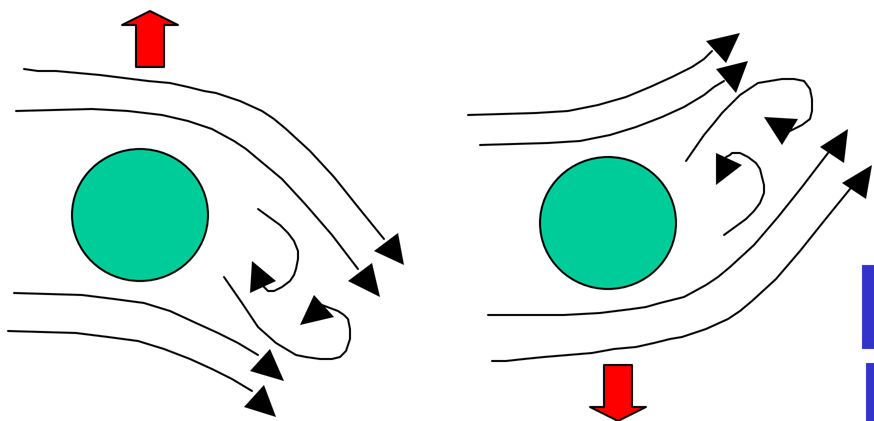
$$R_x = -rV_1^2 A_1 + \dot{m}_2 V_1 + \int_0^1 r(20+30y)^2 dA + \int_1^H rV_1^2 dA$$

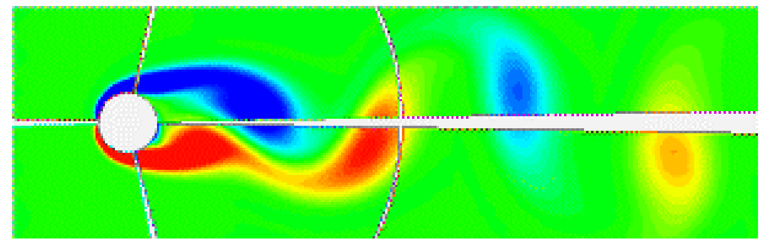
$$= -rV_1^2(1)(H) + \dot{m}_2 V_1 + r[400y + 600y^2 + 300y^3]\Big|_0^1 + rV_1^2(1)(H-1)$$

$$= (18)(50) + (1.2)[400 + 600 + 300] - (1.2)(50)^2(1) = -540(N)$$

Lift and Drag Forces

The force acting on the cylinder by the fluid is equal in magnitude and opposite in direction: $K_x = -R_x = 540(N)$. Drag force is in the positive x direction.





MOVIE (NCSC University of Illinois)

PIV MOVIE

Periodic shedding of vortices into the wake generates alternative up- and down-wash as shown. Consequently, oscillatory loading will be exerted on the cylinder.