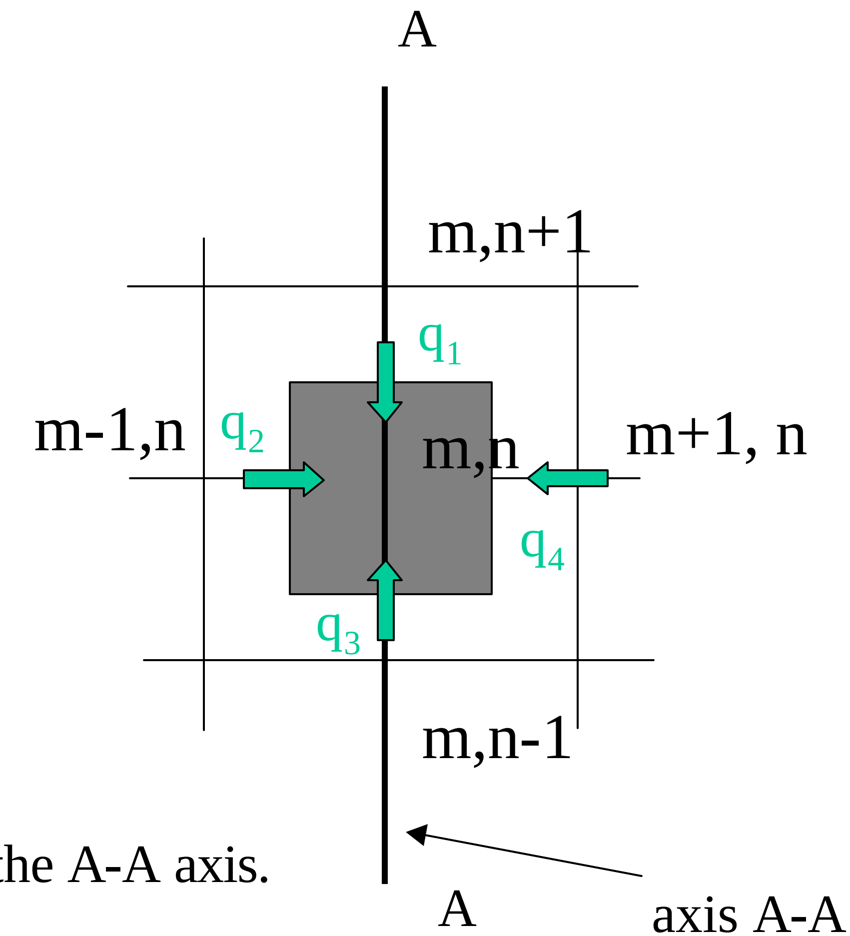
Numerical Method (Special Cases)

For all the special cases discussed in the following, the derivation will be based on the standard nodal point configuration as shown to the right.



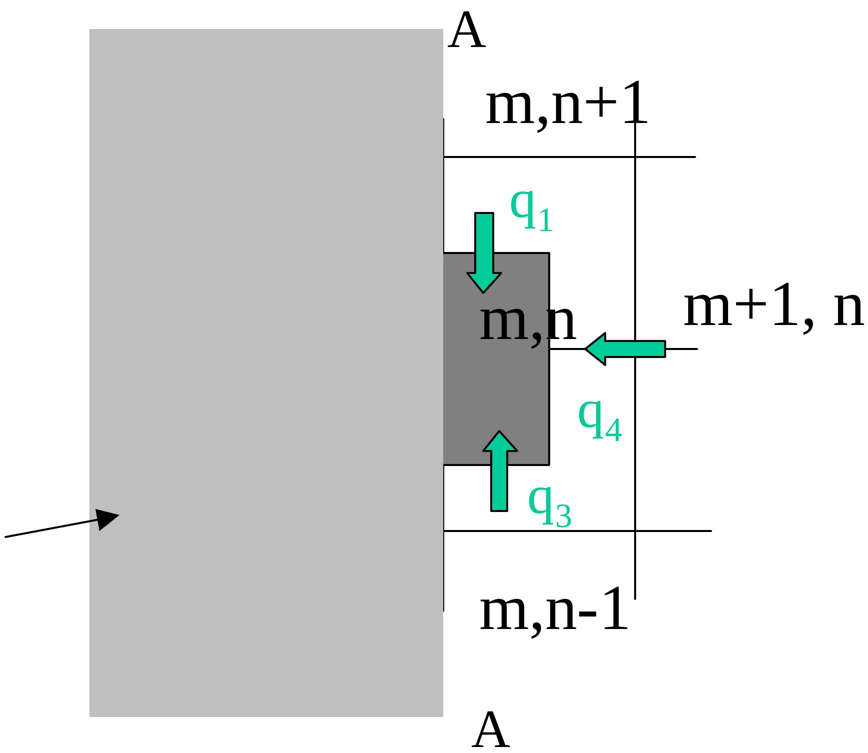
o Symmetric case: symmetrical relative to the A-A axis.
In this case, $T_{m-1,n}=T_{m+1,n}$
Therefore the standard nodal equation can be written as

$$T_{m+1,n}+T_{m-1,n}+T_{m,n+1}+T_{m,n-1}-4T_{m,n}$$
$$=2T_{m+1,n}+T_{m,n+1}+T_{m,n-1}-4T_{m,n}=0$$

Special cases (cont.)

- Insulated surface case: If the axis A-A is an insulated wall, therefore there is no heat transfer across A-A. Also, the surface area for q_1 and q_3 is only half of their original value. Write the energy balance equation (q_2=0):

Insulated surface



$$q_1 + q_3 + q_4 = 0$$

$$k\left(\frac{\Delta x}{2}\right)\frac{T_{m,n+1} - T_{m,n}}{\Delta y} + k\left(\frac{\Delta x}{2}\right)\frac{T_{m,n-1} - T_{m,n}}{\Delta y} + k\Delta y\frac{T_{m+1,n} - T_{m,n}}{\Delta x} = 0$$

$$2T_{m+1,n} + T_{m,n+1} + T_{m,n-1} - 4T_{m,n} = 0$$

This equation is identical to the symmetrical case discussed previously.

Special cases (cont.)

- With internal generation G=gV where g is the power generated per unit volume (W/m^3). Based on the energy balance concept:

$$q_1 + q_2 + q_3 + q_4 + G$$

$$q_1 + q_2 + q_3 + q_4 + g(\Delta x)(\Delta y)(1) = 0$$

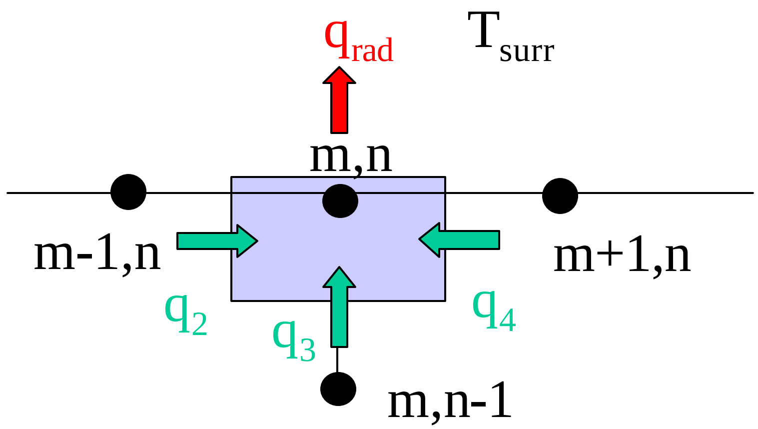
Use 1 to represent the dimension along the z-direction.

$$k(T_{m+1,n} + T_{m-1,n} + T_{m,n+1} + T_{m,n-1} - 4T_{m,n}) + g\left(\Delta x\right)^2 = 0$$

$$T_{m+1,n} + T_{m-1,n} + T_{m,n+1} + T_{m,n-1} - 4T_{m,n} + \frac{g(\Delta x)^2}{k} = 0$$

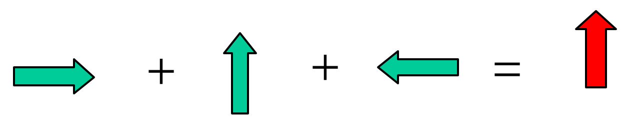
Special cases (cont.)

- Radiation heat exchange with respect to the surrounding (assume no convection, no generation to simplify the derivation).
Given surface emissivity ε, surrounding temperature T_{surr}.



From energy balance concept:

$q_2+q_3+q_4=q_{rad}$



$$k\left(\frac{\Delta y}{2}\right)\frac{T_{m-1,n}-T_{m,n}}{\Delta x}+k(\Delta x)\frac{T_{m,n-1}-T_{m,n}}{\Delta y}+k\left(\frac{\Delta y}{2}\right)\frac{T_{m+1,n}-T_{m,n}}{\Delta x}=es\,(\Delta x)\left(T_{m,n}{}^4-T_{surr}{}^4\right)$$

$$k\left(T_{m-1,n}+T_{m+1,n}+2T_{m,n-1}-4T_{m,n}\right)=2es\,(\Delta x)\left(T_{m,n}^4-T_{surr}^4\right)$$

$$T_{m-1,n}+T_{m+1,n}+2T_{m,n-1}-4T_{m,n}-\frac{2es\,(\Delta x)}{k}T_{m,n}^4=-2\frac{es\,(\Delta x)}{k}T_{surr}^4$$

Non-linear term, can solve using the iteration method