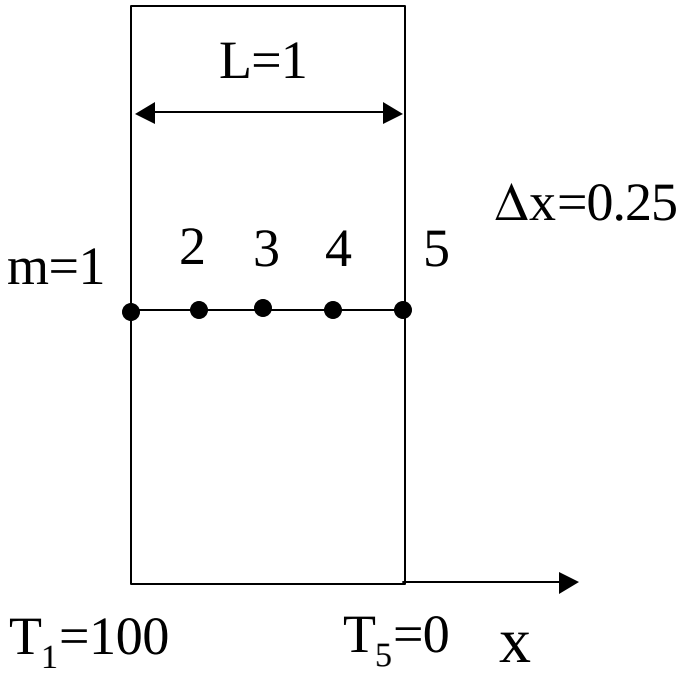
Numerical Examples

Example: Find the system of finite difference equations for the following problem. Steady state heat conduction through a plane wall with no internal generation.



Heat Equation: $\nabla^2 T = \dfrac{\partial^2 T}{\partial x^2} = 0$

Finite difference approximation:

$$\left.\frac{\partial^2 T}{\partial x^2}\right|_m \approx \frac{1}{(\Delta x)^2}\left[T_{m+1} + T_{m-1} - 2T_m\right]$$

For interior nodal points: m=2, 3 & 4

$$T_{m+1} + T_{m-1} - 2T_m = 0$$

Numerical Examples (cont.)

For exterior (boundary) points: m=1 & 5

$T_1 = 100, \; T_5 = 0$

The system of equations:

$$
\begin{aligned}
T_1 + \quad 0 + \quad 0 + \quad 0 + \quad 0 &= 100 \\
T_1 - \; 2T_2 + \; T_3 + \quad 0 + \quad 0 &= 0 \\
0 + \quad T_2 - \; 2T_3 + \; T_4 + \quad 0 &= 0 \\
0 + \quad 0 + \quad T_3 - \; 2T_4 + \; T_5 &= 0 \\
0 + \quad 0 + \quad 0 + \quad 0 + \; T_5 &= 0
\end{aligned}
$$

$$
\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad [\mathbf{A}][\mathbf{T}] = [\mathbf{C}]
$$

MathCAD Solution

Solve the system of equations using matrix inversion function in MathCAD. $[\mathbf{A}][\mathbf{T}]=[\mathbf{C}]$, $[\mathbf{T}]=[\mathbf{A}]^{-1}[\mathbf{C}]$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 100 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad A^{-1}$$

Type A^-1 in MathCAD
Ainv is generated by typing A.inv

$$C$$

Type A.int*C in MathCAD to perform matrix product

$$\begin{bmatrix} 100 \\ 75 \\ 50 \\ 25 \\ 0 \end{bmatrix}$$

Type T= in MathCAD to see the solution vector