# Reynolds Analogy

It can be shown that, under specific conditions (no external pressure gradient and Prandtle number equals to one), the momentum and heat transfers can be related. The momentum transfer of fluid passing a flat plate can be characterized by the skin friction coefficient, $C_f$. The heat transfer between the plate and the flow can be characterized by the Nusselt number, Nu. They can be related as:

$$C_f \frac{\mathrm{Re}}{2} = Nu, \text{ where } \mathrm{C_f} = \frac{t_w}{\frac{1}{2} r V^2}, Nu = \frac{hL}{k_f}.$$

$$\text{Define Stanton number St}: \quad \mathrm{St} = \frac{\mathrm{h}}{r\mathrm{VCp}} = \frac{Nu}{\mathrm{Re\,Pr}}$$

$$\text{The analogy becomes}: \frac{\mathrm{C_f}}{2} = St$$

# Reynolds Analogy (cont.)

The Reynolds analogy related the flow parameters to the thermal parameters. If a given flow field can be determined, the heat transfer characteristics can be found by using the Reynolds analogy.

A modified Reynolds analogy has been obtained to take into consideration of the fact that Prandtle number Pr is usually not equal to one:

$$\frac{C_f}{2} = St\,\mathrm{Pr}^{2/3}, \quad \text{for } 0.6 < \mathrm{Pr} < 60.$$

Note: The Reynolds analogy should only be used when the pressure gradient is zero. However, in turbulent flow, this condition is not that important. Therefore, the analogy can be applied even when the pressure gradient is nonzero for turbulent flows.

# Laminar Boundary Layer

As discussed earlier, laminar boundary layer solution can be calculated analytically and we call the solution the Blasius solution. For the flow over a flat plate with a free-stream velocity $U_\infty$. The wall shear stress is:

$$t_w = m \left. \frac{\partial u}{\partial y} \right|_{y=0} = 0.332 U_\infty \sqrt{\frac{r m U_\infty}{x}}$$

The local friction coefficient is :

$$C_f = \frac{t_w}{\frac{1}{2} r U_\infty^2} = 0.664 \, \mathrm{Re}_x^{-1/2} \, .$$

From modified Reynolds analogy :

$$\frac{C_f}{2} = St \, \mathrm{Pr}^{2/3} = \frac{Nu}{\mathrm{Re}\,\mathrm{Pr}} \mathrm{Pr}^{2/3}, \quad Nu = \frac{C_f}{2} \mathrm{Re}\,\mathrm{Pr}^{1/3} = 0.332 \, \mathrm{Re}_x^{1/2} \, \mathrm{Pr}^{1/3}$$

# Reynolds Analogy (cont.)

To obtain the averaged convection coefficient, we can integrate the local coefficient along the plate.

$$\bar{h} = \frac{1}{x}\int_0^x h_x dx = 0.332(\frac{k_f}{x})\Pr^{1/3}(\frac{U_\infty}{\nu})^{1/2}\int_0^x \frac{dx}{x^{1/2}}$$

$$\overline{Nu} = \frac{\bar{h}x}{k} = 0.664\,\mathrm{Re}_x^{1/2}\Pr^{1/3}$$