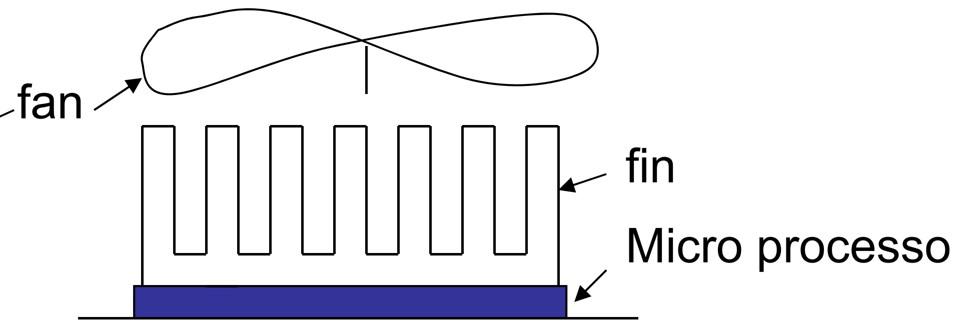
3. Extended surface (a fin) heat transfer

Objectives :

1. **To examine heat transfer in a single cylindrical extended surface (a fin) in free and forced convection**
2. **To develop an understanding of fin effectiveness and the parameters which influence it.**



Fins

Extended surfaces or Fins are generally used to enhance convective heat transfer rate between a solid and the surrounding fluid.

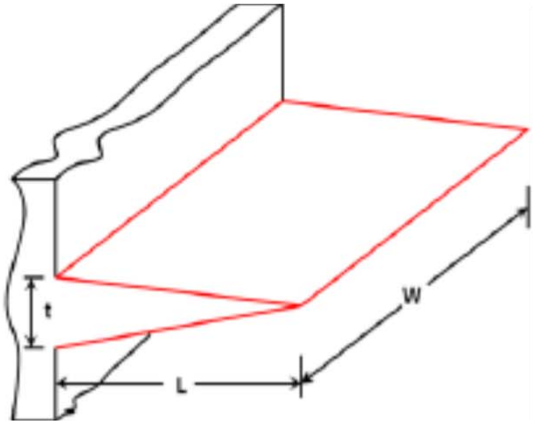
Simply put: A fin extends the surface area of heat transfer.

The fin material generally has a high thermal conductivity which is exposed to a flowing fluid.

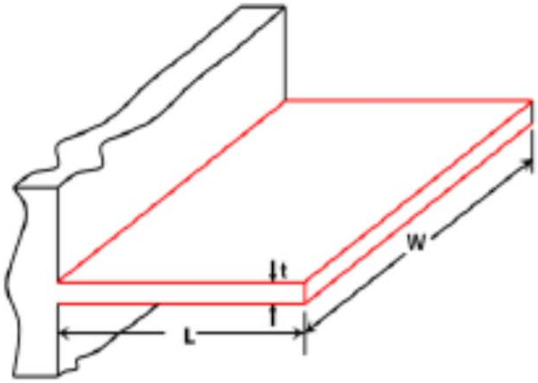
Fins are often seen in electrical appliances and electronics such as on computer processors and power supplies and industrial applications such as heat exchangers and substation transformers

Fins – Different configurations

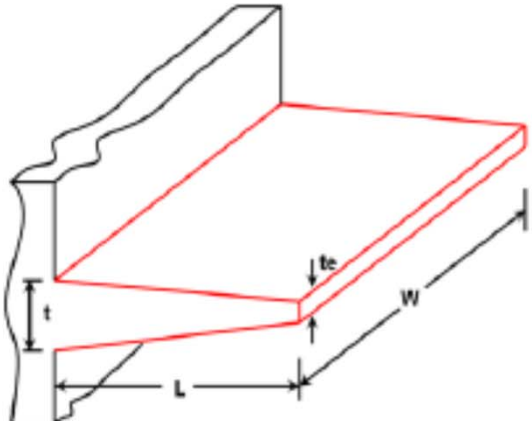
Triangular fin

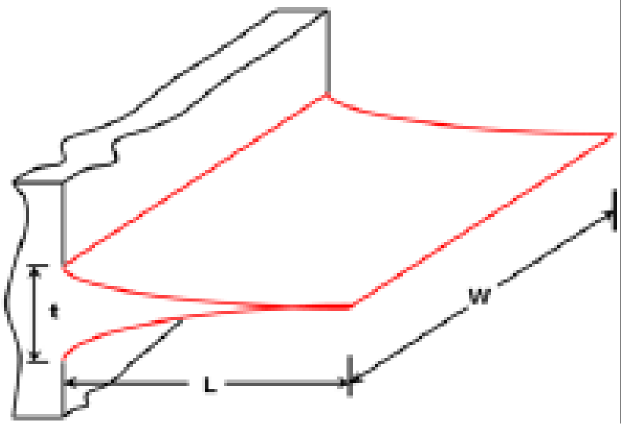


Rectangular fin

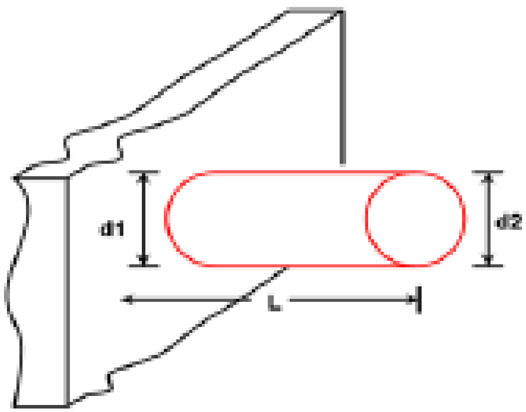


Trapezoidal fin





Parabolic fin



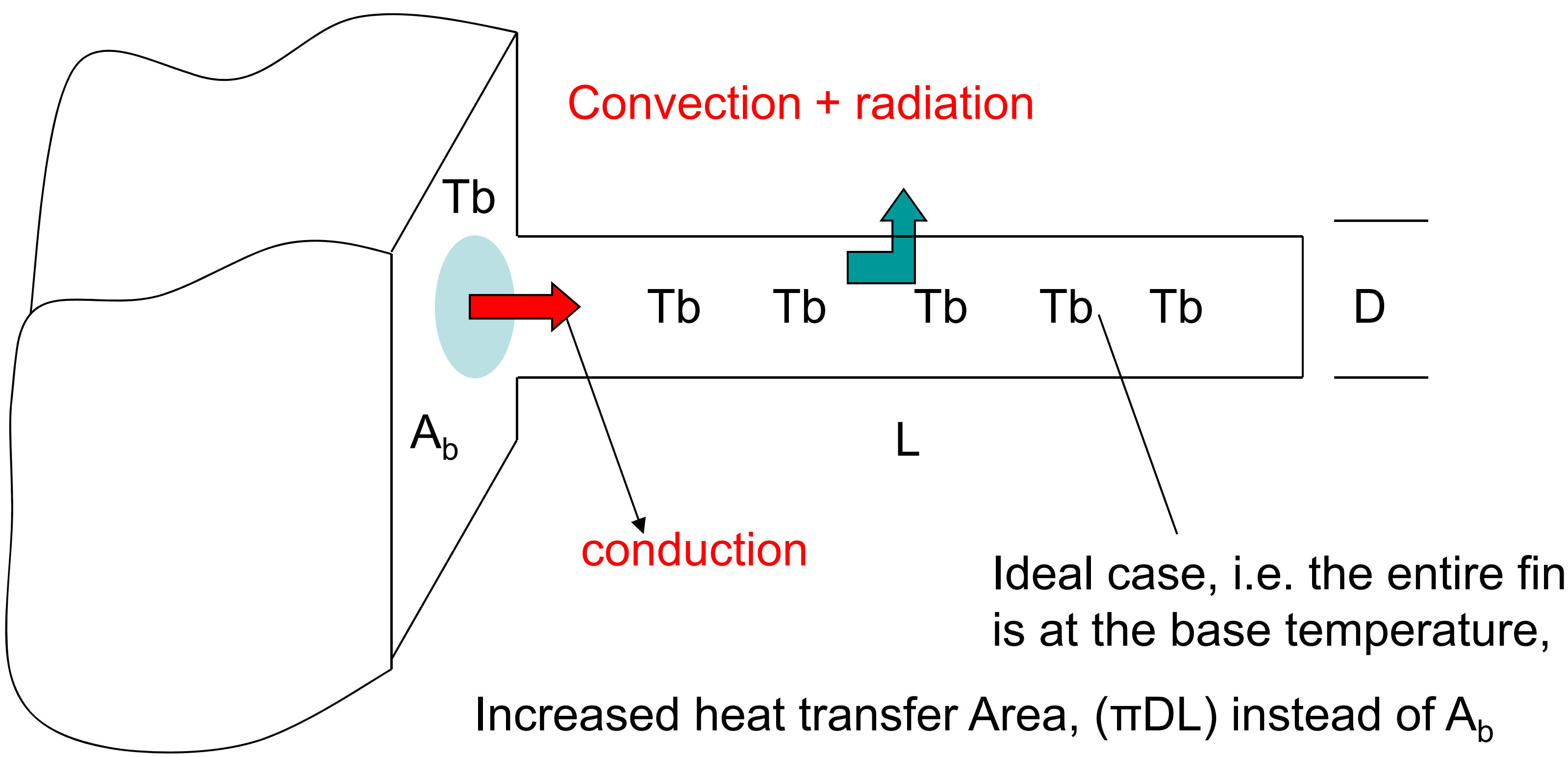
Cylindrical fin

Within a fin, heat is transferred via conduction.

Heat transferred to surrounding via convection and radiation

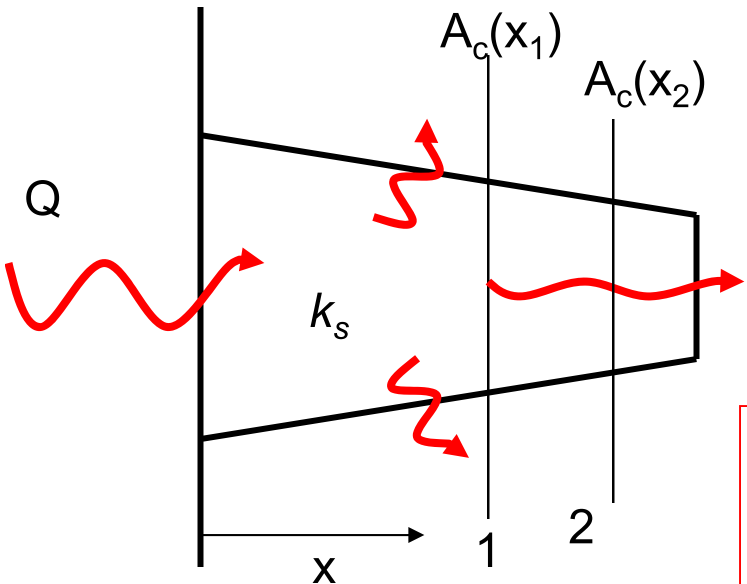
In general, we would like the entire fin to be @ the base temperature

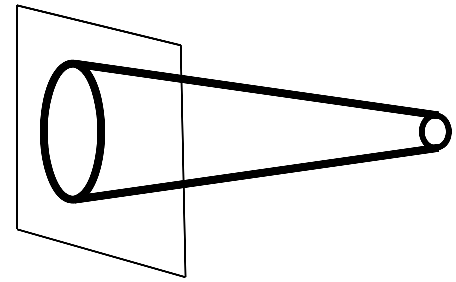
WHY?

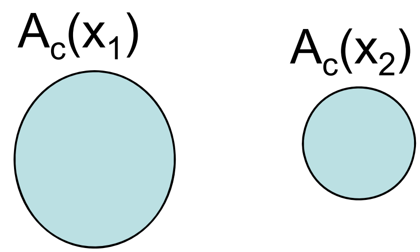


Increased heat transfer Area, (πDL) instead of A_b

Differential heat equation for a fin







$A_c(x)$ is cross sectional area
Heat conducted through this area inside the fin

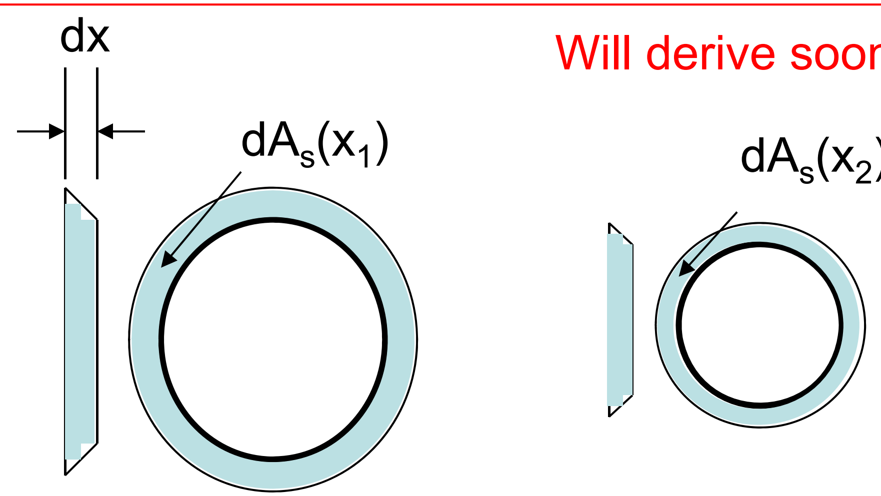
$$\frac{d^2T}{dx^2} + \left(\frac{1}{A_c}\frac{dA_c}{dx}\right)\frac{dT}{dx} - \left(\frac{1}{A_c}\frac{h}{k_s}\frac{dA_s}{dx}\right)(T - T_\infty) = 0$$

Will derive soon!

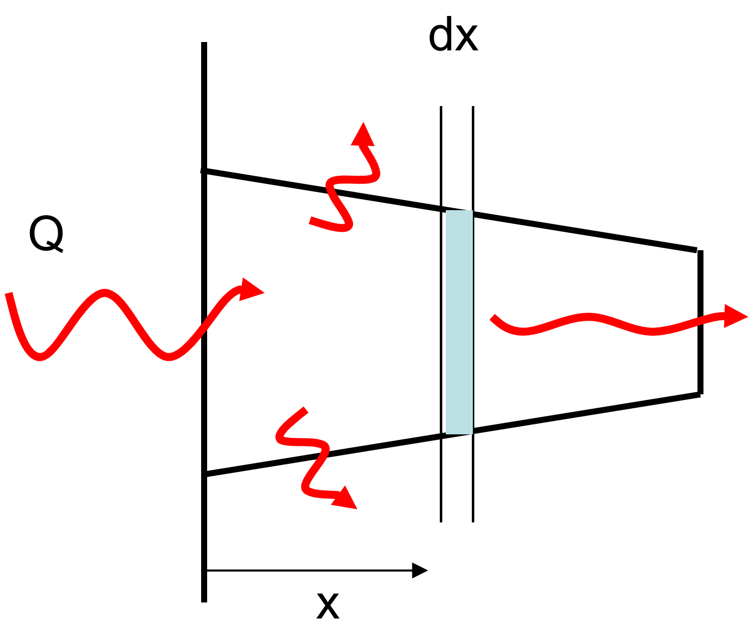
k_s thermal conductivity of solid

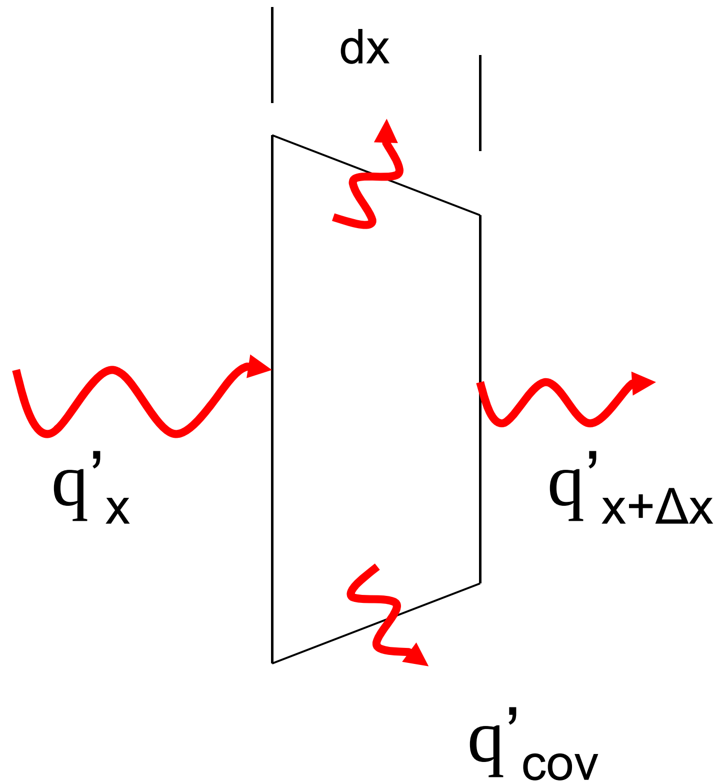
Assumptions (V. IMPORTANT)

1. **Steady state**
2. **One dimensional**
3. **Uniform convective heat transfer coefficient**
4. **Constant thermal conductivity**



$A_s(x)$ is surface area
Heat convected to ambience through this area





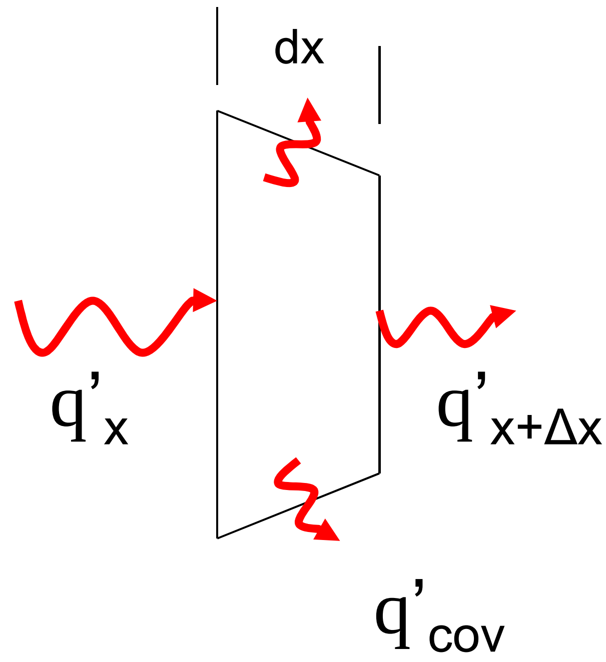
<u>No heat generation</u>
<u>Steady state</u>
<u>Radiation neglected</u>

Energy balance will lead to

$$q'_x = q'_{x+\Delta x} + q'_{con}$$

$$q'_x = -k_s A_c \frac{dT}{dx}$$

$$q'_{x+\Delta x} = q'_x + \frac{\partial q'_x}{\partial x} dx$$



$$q'_{x+\Delta x} = -k_s A_c \frac{dT}{dx} + \frac{d}{dx}\left(-k_s A_c \frac{dT}{dx}\right) dx$$

$$q'_{con} = h(dA_s)(T - T_\infty) \qquad dA_s = P dx$$

$$q'_x = q'_{x+\Delta x} + q'_{con}$$

$$\Rightarrow -k_s A_c \frac{dT}{dx} = -k_s A_c \frac{dT}{dx} + \frac{d}{dx}\left(-k_s A_c \frac{dT}{dx}\right) dx + h(dA_s)(T - T_\infty)$$

$$-k_s A_c \frac{dT}{dx} = -k_s A_c \frac{dT}{dx} + \frac{d}{dx}\left(-k_s A_c \frac{dT}{dx}\right)dx + h(dA_s)(T - T_\infty)$$

$$\frac{d}{dx}\left(k_s A_c \frac{dT}{dx}\right)dx - h(dA_s)(T - T_\infty) = 0$$

$$\frac{d}{dx}\left(A_c \frac{dT}{dx}\right) - \frac{h}{k_s}\left(\frac{dA_s}{dx}\right)(T - T_\infty) = 0$$

$$A_c \frac{d^2T}{dx^2} + \frac{dT}{dx}\frac{dA_c}{dx} - \frac{h}{k_s}\left(\frac{dA_s}{dx}\right)(T - T_\infty) = 0$$

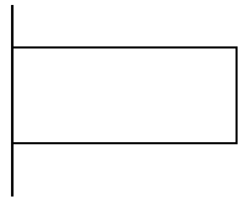
$$\boxed{\frac{d^2T}{dx^2} + \frac{1}{A_c}\frac{dA_c}{dx}\frac{dT}{dx} - \frac{1}{A_c}\frac{h}{k_s}\frac{dA_s}{dx}(T - T_\infty) = 0}$$

Derived

SIMPLIFICATIONS

$$\frac{d^2T}{dx^2} + \frac{1}{A_c}\frac{dA_c}{dx}\frac{dT}{dx} - \frac{1}{A_c}\frac{h}{k_s}\frac{dA_s}{dx}(T - T_\infty) = 0$$

Assume uniform cross sectional area, i.e A_c is constant



$$dA_s = P dx$$

$$\frac{dA_s}{dx} = P$$

P is the perimeter

$$\frac{d^2T}{dx^2} - \frac{1}{A_c}\frac{hP}{k_s}(T - T_\infty) = 0$$

$$\frac{d^2T}{dx^2} - \frac{1}{A_c}\frac{hP}{k_s}(T - T_\infty) = 0$$

$$Define \;\; \theta = T - T_\infty$$

$$m = \sqrt{\frac{hP}{k_s A_c}}$$

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

A second order ordinary DE with constant coefficient

Solution :

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

$$\Rightarrow T - T_\infty = C_1 e^{mx} + C_2 e^{-mx}$$

$$C_1 \;\; and \;\; C_2 \;\; depends \;\; upon \;\; the \;\; B.Cs$$

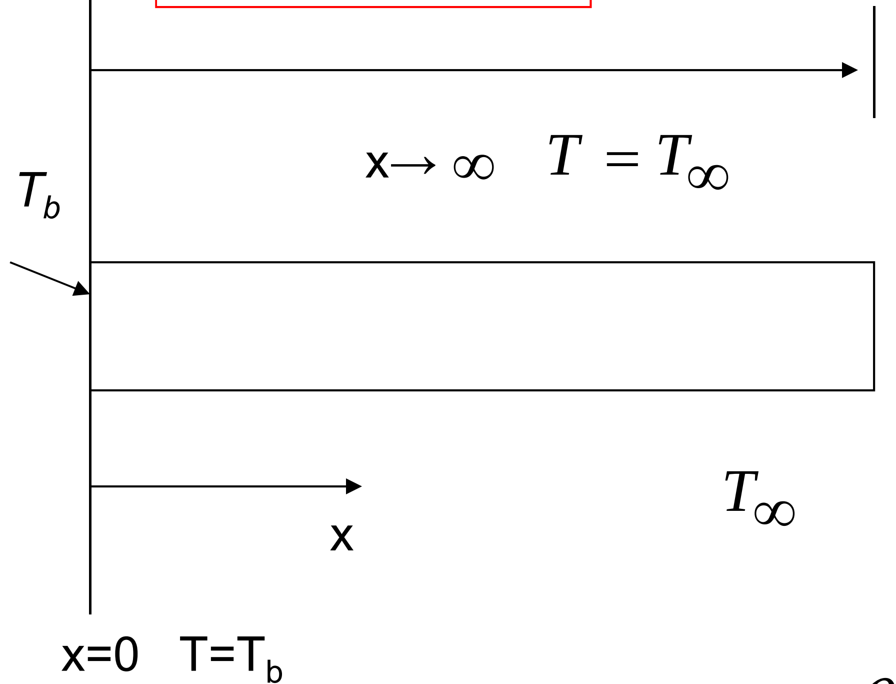
Effect of Boundary Conditions

<u>**Consider an infinite long fin**</u> ***x*→∞ , *T* = *T*∞**

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

Need two BCs for x



***x* = 0, $T=T_b$ $\theta = T_b - T_\infty = \theta_b$**

***x*→∞, $T=T_\infty$ $\theta = T_\infty - T_\infty = 0$**

Second BC → $C_1=0$

First BC→C2= θ_b

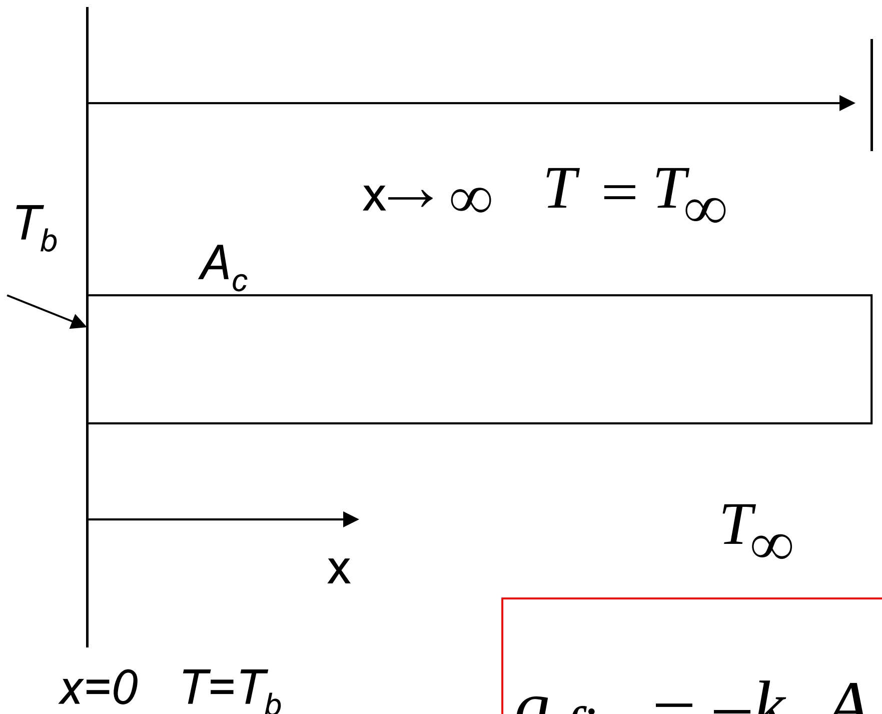
Solution is $$\theta(x) = \theta_b e^{-mx}$$

$$T(x) - T_\infty = (T_b - T_\infty)e^{-mx}$$

$$T(x) = T_\infty + (T_b - T_\infty)e^{-mx}$$

(a) Heat transfer from an infinitely long fin

How can we estimate the total Heat Transfer?



$$\theta(x) = \theta_b e^{-mx}$$

$$\theta = T - T_\infty$$

$$T(x) = T_\infty + (T_b - T_\infty)e^{-mx}$$

$$q_{fin} = -k_s A_c \left.\frac{d\theta}{dx}\right|_{x=0} = \int_{x=0}^{x=L} h\theta P dx$$

$$q_{fin} = -k_s A_c \left.\frac{d\theta}{dx}\right|_{x=0}$$

$$\theta(x) = \theta_b e^{-mx}$$

$$q_{fin} = -k_s A_c \left. \frac{d(\theta_b e^{-mx})}{dx} \right|_{x=0}$$

We have the temperature distribution

$$\theta(x) = \theta_b e^{-mx}$$

$$q_{fin} = k_s A_c \theta_b m \left| e^{-mx} \right|_{x=0}$$

where

$$m = \sqrt{\frac{hP}{k_s A_c}}$$

$$q_{fin} = k_s A_c \theta_b m$$

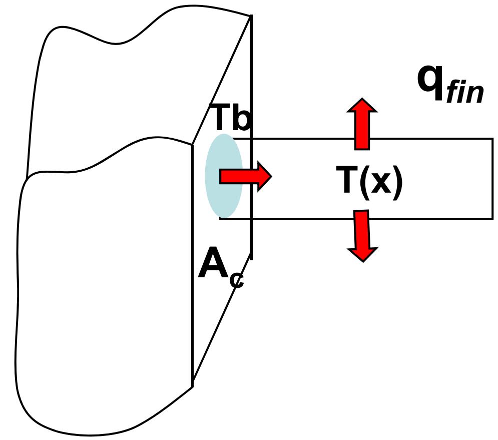
$$q_{fin} = k_s A_c \theta_b \sqrt{\frac{hP}{k_s A_c}}$$

$$\boxed{q_{fin} = \sqrt{hPk_s A_c}\ \ \theta_b}$$

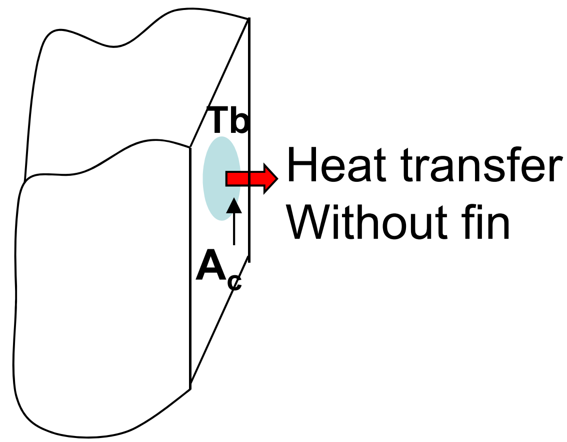
This is heat transfer from an infinitely long fin

$$\boxed{q_{fin} = \sqrt{hPk_s A_c}\ \ (T_b - T_\infty)}$$

Fin effectiveness



- To use / or not use a fin?
- Need to compare q'_{fin} with heat transfer without fin
- A parameter called fin effectiveness ε_f is defined as



$$\varepsilon_f = \frac{q_{fin}}{hA_c(T_b - T_\infty)}$$

Substitute the appropriate q_{fin} (depends on B.Cs) and find ε_f

To justify the use of fin, $\varepsilon_f \geq 2$

For an infinite long fin

$$q_{fin} = \sqrt{hPkA_c}\,(T_b - T_\infty)$$

$$\varepsilon_f = \frac{\sqrt{hPkA_c}\,(T_b - T_\infty)}{hA_c(T_b - T_\infty)}$$

$$\varepsilon_f = \sqrt{\frac{kP}{hA_c}}$$

For an infinitely long fin

Which Paramters influence fin effectiveness and how?

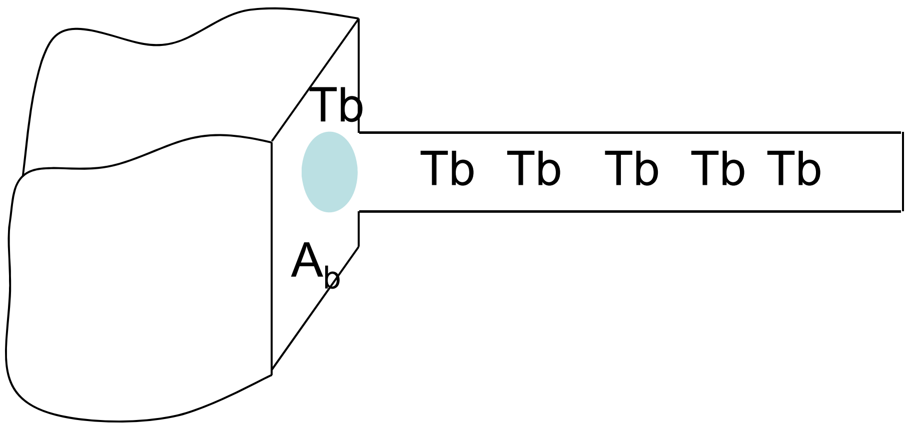
1. **To increase ε_f, need to increase (k/h)**
2. **Increase P/A_c - i. e perimeter to cross sectional area ---use thin fins**
3. **Hence in heat exchanger, we this, closely packed thin fins**
4. **Fins should be used when/where *h* is small. Commonly used on gas side of liquid-gas heat exchangers.**

Fin efficiency

$$\eta_f = \frac{q_f}{q_{\max}}$$

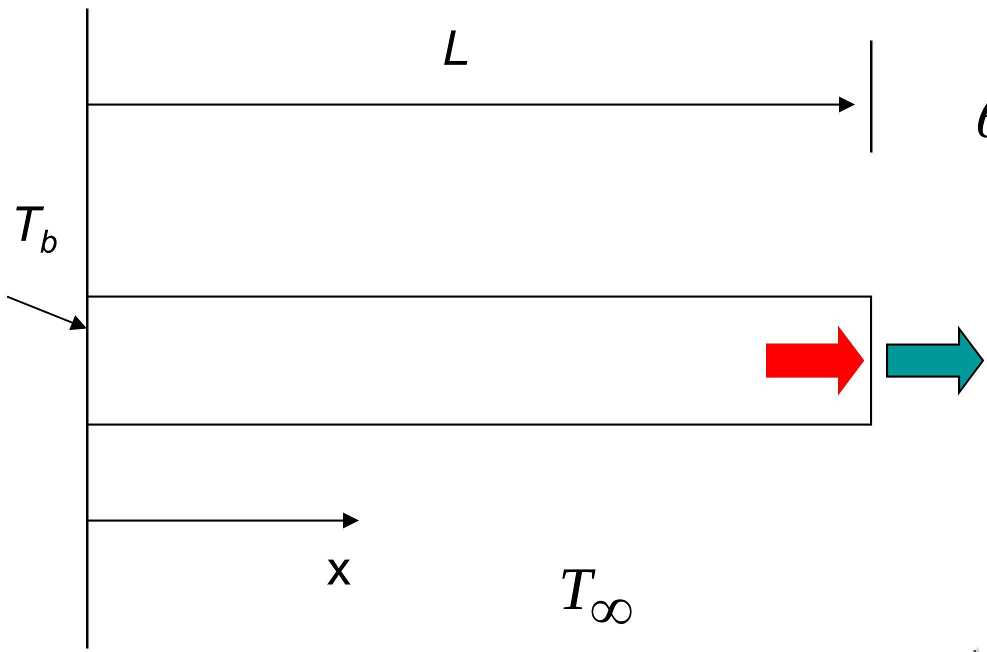
$$\eta_f = \frac{q_f}{hA_f\theta_b}$$

Where q_{max} = Theoretical maximum heat transfer if the entire fin is at the Base temperature



Other possible boundary condition at the tip

a) Convective BC at the tip



x=0 T=T_b θ= θ_b

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

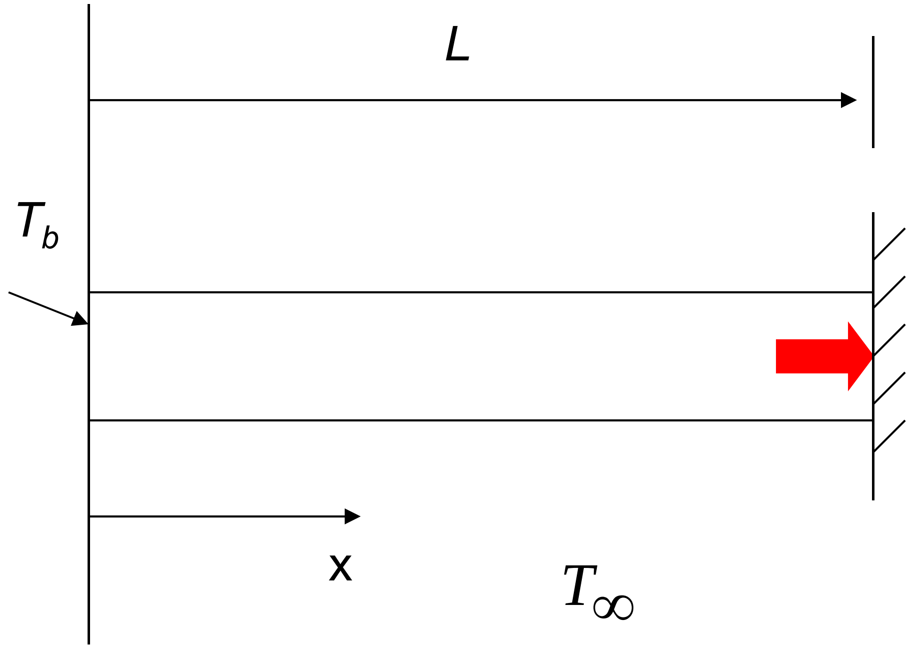
$$\theta_{x=0} = \theta_b$$

$$h\theta(L) = -k\left.\frac{d\theta}{dx}\right|_{x=L}$$

$$\frac{\theta}{\theta_b} = \frac{\cosh m(L-x) + \left(\frac{h}{mk}\right)\sinh m(L-x)}{\cosh mL + \left(\frac{h}{mk}\right)\sinh mL}$$

$$q_{fin} = \sqrt{hPkA_c}\,\theta_b \frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$$

b) Adiabatic BC at the tip



x=0 $T=T_b$ $\theta = \theta_b$

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

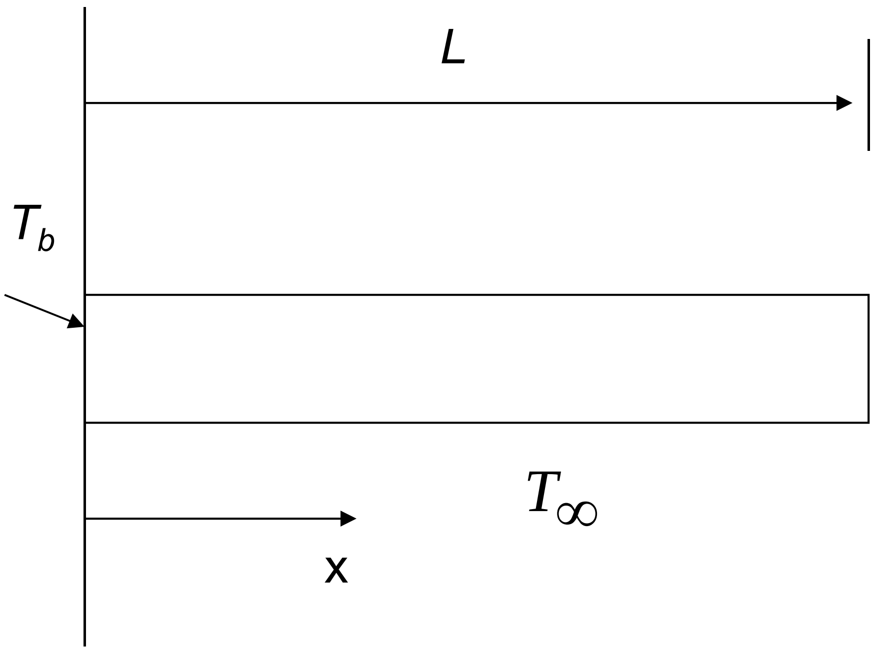
$$\theta_{x=0} = \theta_b$$

$$\left.\frac{d\theta}{dx}\right|_{x=L} = 0$$

$$\frac{\theta}{\theta_b} = \frac{\cosh m(L-x)}{\cosh mL}$$

$$q_{fin} = \sqrt{hPkA_c}\,\theta_b \tanh mL$$

c) Prescribed temperature at the tip



$\theta_{x=L} = \theta_L$

$x=0 \quad T=T_b \quad \theta = \theta_b$

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

$$\theta_{x=0} = \theta_b$$

$$\frac{\theta}{\theta_b} = \frac{\frac{\theta_L}{\theta_b}\sinh mx + \sinh m(L-x)}{\sinh mL}$$

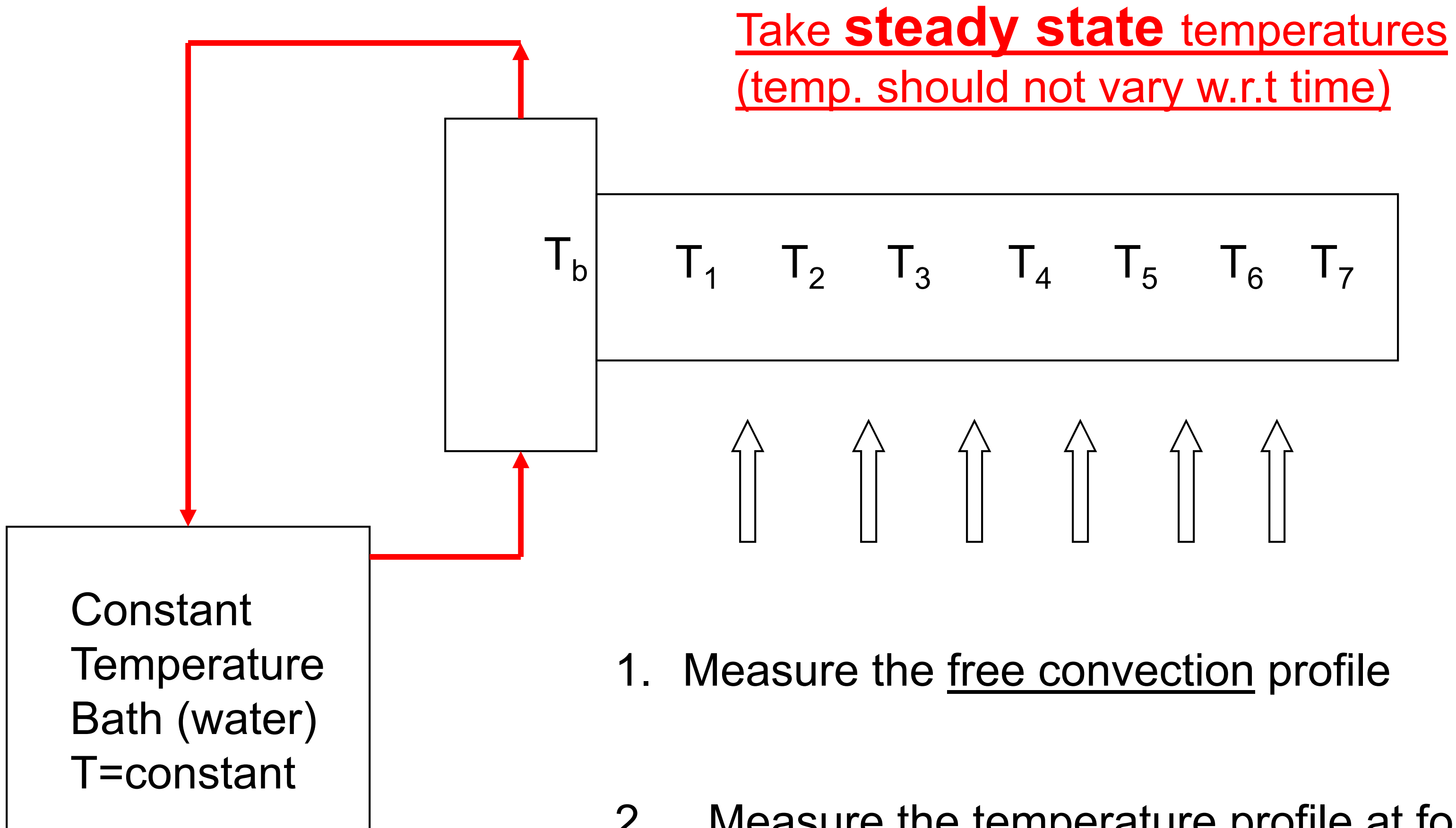
$$\mathbf{q_{fin}} = \sqrt{hPkA_c}\,\theta_b \frac{\cosh mL - \frac{\theta_L}{\theta_b}}{\sinh mL}$$

Case	Tip Condition (x = L)	Temperature Distribution θ/θ_b	Fin Heat Transfer Rate, q_f
A	Convection $h\theta(L) = -k \left.\frac{d\theta}{dx}\right\|_{x=L}$	$\frac{\cosh[m(L-x)]+\left(\frac{h}{mk}\right)\sinh[m(L-x)]}{\cosh mL+\left(\frac{h}{mk}\right)\sinh mL}$	$M\frac{\sinh mL+\left(\frac{h}{mk}\right)\cosh mL}{\cosh mL+\left(\frac{h}{mk}\right)\sinh mL}$
B	Adiabatic $\left.\frac{d\theta}{dx}\right\|_{x=L} = 0$	$\frac{\cosh[m(L-x)]}{\cosh mL}$	$M \tanh mL$
C	Prescribed Temp. $\theta(L) = \theta_L$	$\frac{\left(\frac{\theta_L}{\theta_b}\right)\sinh mx+\sinh[m(L-x)]}{\sinh mL}$	$M\frac{\cosh mL-\frac{\theta_L}{\theta_b}}{\sinh mL}$
D	Infinite fin (L $\rightarrow \infty$ $\theta(L) = 0$	e^{-mx}	M

$\theta \equiv T - T_\infty$ $\qquad$ $m^2 = \frac{hP}{kA_c}$

$\theta_b = \theta_0 = T_b - T_\infty$ $\qquad$ $M = \sqrt{hPKA_c}\theta_b$

Your experiment

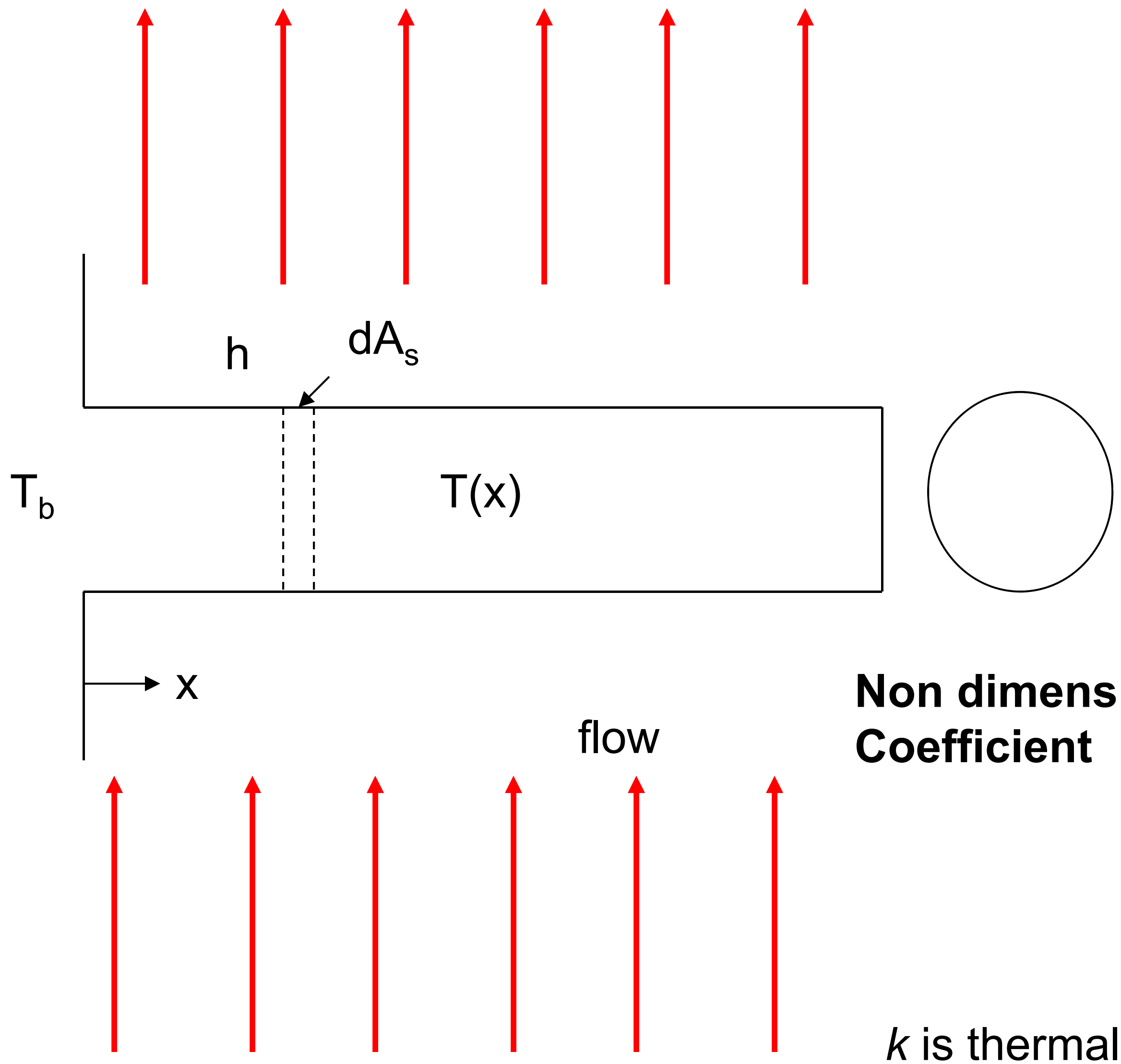


1. Measure the free convection profile

2. Measure the temperature profile at forced convection ambience (use wind tunnel to change the wind speed)

How do you ensure that steady state conditions have been achieved?

Average convection coefficient $\bar{h}$

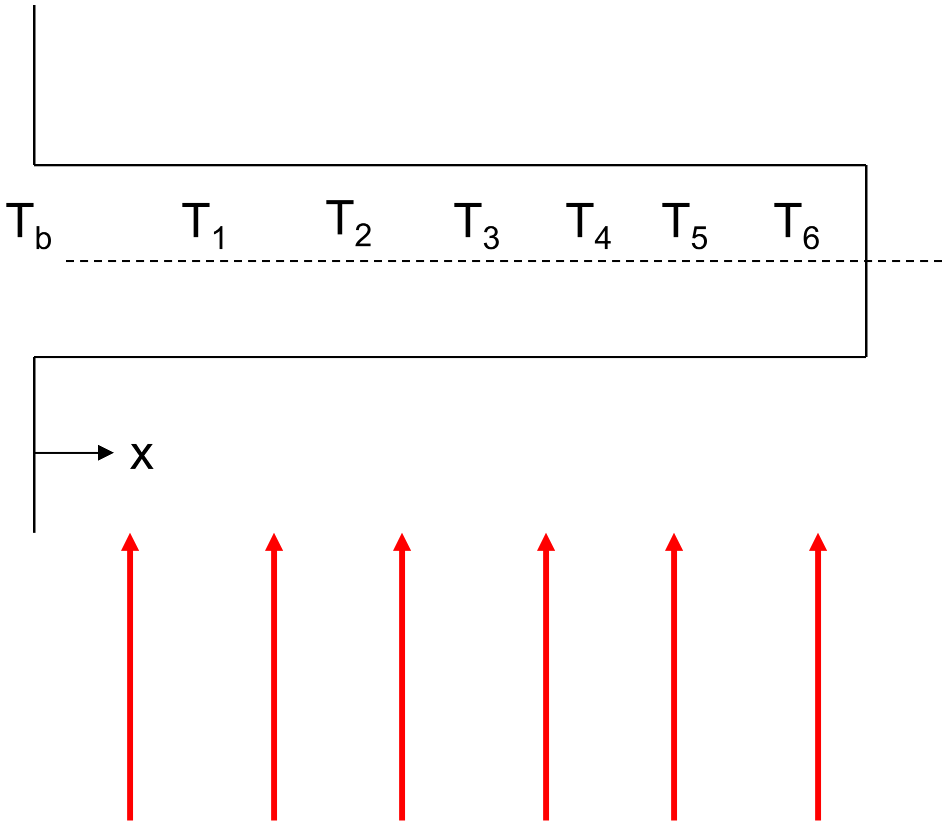


$$\bar{h} = \frac{1}{A_s} \int_{A_S} h\, dA_s$$

Non dimensional convective heat transfer Coefficient

$$\overline{Nu} = \frac{\bar{h} d}{k_f}$$

k is thermal conductivity of fluid and
d is characteristic length, here it is diameter of cylinder



$$q'' = h(T - T_\infty)$$

$$q_{fin} = \int_{area} h(T - T_\infty)$$

$$q_{fin} = \bar{h} \int_{area} (T - T_\infty)$$

$$q_{fin} = \bar{h} \int_0^L (T - T_\infty)dx$$

$$\overline{Nu} = \frac{\bar{h}d}{k_f} \Rightarrow \bar{h}$$