EML 5060 **Homework Set 3** **Fall 2003**

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64	6.59a		Air resistance*#
81	8.28	8.18	Vibrational and growth type[1]#
81	8.23	8.19	Vibrational and growth type#
81	8.24	8.21	Vibrational and growth type#
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96	10.44	10.45	Vibrational and growth, forced#
96	10.46		Vibrational and growth, forced#
96	10.52	10.47	Vibrational and growth, forced#
103	11.9	11.10	Vibrational and growth, forced#
103	11.14		Vibrational and growth, forced#
103	11.26	11.25	Vibrational and growth, forced#
107	12.10	12.11	Vibrational and growth, forced#
122	13.40		Spring mass system*#
198	22.22	22.12	Solve as 22.12 (required)[2]

*: Recommended question. Not required if you know you can do it.

#: Make a graph. For problems with more than one unknown parameter, draw the solutions taking one parameter 1 and the rest 0.

[1]: Second double dot should be single dot.

[2]: Solution appears to be wrong.

Also solve the 4 questions below*:

1. Solve the Cauchy equation

$$x^2y'' + xy' - 4y = \ln x^2$$

by taking $u = \ln|x|$ as the new independent variable. To eliminate x, use the chain rule of differentiation as in

$$y' \equiv \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \frac{dy}{du}\frac{1}{x},$$

and once more to find y'' in terms of dy/du and d^2y/du^2. Please *do not* indicate dy/du also by y'! Solution:

$$y = -\tfrac{1}{2}\ln x + Ax^2 + Bx^{-2}$$

2. Solve the aerodynamically damped spring-mass system

$$\ddot{y} + (\dot{y})^2 + y = 0$$

by taking y as the independent variable and $\dot{y}$ as the dependent variable. To eliminate the remaining dt, (in $\ddot{y} = d\dot{y}/dt$), use the chain rule of differentiation. Solution:

$$\dot{y}^2 = -y + \tfrac{1}{2} + C_0 e^{-2y}, \text{ hence } t = \pm \int \frac{dy}{\sqrt{-y + \frac{1}{2} + C_0 e^{-2y}}}$$

3. Solve the motion of a falling body with aerodynamic drag:

$$\ddot{x} + (\dot{x})^2 = 1.$$

Solution:

$$\dot{x} = \frac{Ce^{2t} - 1}{Ce^{2t} + 1} \qquad x = \ln |Ce^{2t} + 1| - t + D$$

4. Solve the equation for the streamfunction in a Stokes boundary layer:

$$y'' + 2xy' - 2y = 0.$$

Note that $y = x$ is one solution. Solution:

$$y = C_0 x + C_1 x \int \frac{e^{-x^2}}{x^2} dx$$

Also: Make exam 3 of 1998. Give yourself 50 minutes. Include your solutions with homework set I and grade yourself using the solutions on the web after you get it back.