EML 5060 **Homework Set 4** **Fall 2003**

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17	2.19cfg	2.19e	Classification#
17	2.20		Classification#
17	2.21ac	2.21b	Classification: assume $u = u(x, y, z[, t])$#
18	—	2.24	Canonical form#
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18	2.26		Canonical form#
18	2.22bf	2.22d	Characteristics#
18	2.27b	2.27d	2D Canonical form#
18	2.28egj	2.28nml	2D Canonical form#
99	7.27	7.28	Acoustics in a pipe (use two methods)#
99	7.39	7.38	Potential flow inside a cylinder#
98	7.25	7.24	Unidirectional viscous flow#
99	7.35	7.36	Steady supersonic flow#
98	7.20	7.19	Unsteady heat conduction in a bar#
98	7.21	7.22	Unsteady heat conduction in a bar#
99	7.37	7.37	Steady heat conduction in a plate#
00			Unsteady heat conduction in a disk#

#: Make a graph.

[1]: Plug it in.

[2]: No solution of the PDE is needed, you can answer from symmetry.

[3]: The solution for $x^2 + y^2 = 1$ is given. Guess the solution for $x^2 + y^2 < 1$

[4]: Use 3.37 with f nonzero only in a small range $\theta_1 < \theta < \theta_2$ and see what part of the interior becomes nonzero.

[5]: Assume $\nabla^2 u = 0$ instead of 1. Then make a physical argument based on the physical interpretation of steady heat conduction in a circle with a heat flux 2 entering through the perimeter.

[6]: Plug it in.

[7]: See bottom p.47.

[8]: Solutions must be of the form $u = \sin(\alpha\pi x)\sin(\alpha\pi t)$. See when they satisfy the given conditions.

Also solve the following problem:

NO working together on the problem below! If you get stuck, ask the instructor or TA only.

Describe how a spike of heat diffuses out in a disk with constant temperature boundaries. Assume that the initial spike is located a quarter radius away from the center of the disk. Use the separation of variables (eigenfunction expansion) method as used in class for the heat transfer in a disk.

The PDE governing the temperature $u(r, \vartheta, t)$ in the disk is the $2D$ heat equation:

$$u_t = \kappa \nabla^2 u$$

where κ is the heat conduction coefficient. Take the radius of the disk to be r_0, so that the constant temperature boundary satisfies

$$u(r_0, \vartheta, t) = T_0$$

where T_0 is a given constant.

Take the initial spike of heat to be a delta function:

$$u(r, \vartheta, 0) = \delta(r - \tfrac{1}{4}r_0)\delta(\vartheta)$$

The properties of the delta function will allow you to do the orthogonality integrals analytically. You can integrate the delta function analytically versus any function $\phi(r, \vartheta)$ you want; the result will always be

$$\int\int \phi(r, \vartheta)\delta(r - \tfrac{1}{4}r_0)\delta(\vartheta)\mathrm{d}r\mathrm{d}\vartheta = \phi(\tfrac{1}{4}, 0)$$

Follow the general steps of the problem we did in class, but make appropriate changes. Explain what you are doing at every step. Work everything out as far as possible, which means completely.

In particular, include a table of the values of μ_{nm} for $n \leq 1, m \leq 2$ and a table of the values of f^i_{nm} for $n \leq 1, m \leq 2, i \leq 2$. (When getting these actual μ_{nm} and f^i_{nm} values, substitute $r_0 = 3$ and $T_0 = 0$ in your general expressions.)

Use these values to approximate the temperature at the center of the disk as a function of time in terms of κ.

Unlike you might expect, this expression will become more and more accurate as time progesses. Why?