# Introduction

Multiple integrals:

- Areas (cost, ...):

$$\mathrm{d}A = \mathrm{d}x\,\mathrm{d}y \qquad \mathrm{d}A = \rho\,\mathrm{d}\rho\,\mathrm{d}\theta$$

- Volumes (weight, ...):

$$\mathrm{d}V = \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z \qquad \mathrm{d}V = \rho\,\mathrm{d}\rho\,\mathrm{d}\theta\,\mathrm{d}z \qquad \mathrm{d}V = r^2\sin\phi\,\mathrm{d}r\,\mathrm{d}\phi\,\mathrm{d}\theta$$

- Centroids (center of gravity, center of pressure, ...)

$$\bar{x} = \int x\,\mathrm{d}A \Big/ \int \mathrm{d}A \qquad \bar{x} = \int x\,\mathrm{d}V \Big/ \int \mathrm{d}V$$

- Moments of inertia (solid body dynamics, center of pressure, ...)

$$I_x = \int y^2\,\mathrm{d}A \qquad I_0 = \int x^2 + y^2\,\mathrm{d}A$$

$$I_x = \int y^2 + z^2\,\mathrm{d}V \qquad I_{xy} = -\int xy\,\mathrm{d}V$$

- ...

Notes:

- Draw the region to be integrated over.
- When integrating, say $\int\int\int f(a,b,c)\ \mathrm{d}a\,\mathrm{d}b\,\mathrm{d}c$, you have to decide whether you want to do $a$, $b$, or $c$ first.
- Usually, you do the coordinate with the easiest limits of integration first.
- If you decide to do, say, $b$ first, ($\int_{b_1}^{b_2} f(a,b,c)\ \mathrm{d}b$ first), the limits of integration $b_1$ and $b_2$ must be identified from the graph at *arbitrary* $a$ and $c$, and are normally functions of $a$ and $c$: $b_1 = b_1(a,c)$, $b_2 = b_2(a,c)$.
- After integrating over, say, $b$, the remaining double integral should no longer depend on $b$ in any way. Nor does the region of integration: redraw it without the $b$ coordinate. Then integrate over the next easiest coordinate in the same way.