Page 250, #10(z)

1 p250, #10(z), §1 Asked

Asked:

$$\lim_{x\to 0}(x-\arcsin x)\csc^3 x \tag{1}$$

2 p250, #10(z), §2 Procedure

L'Hopital:

$$\lim_{x\to 0}(x-\arcsin x)\csc^3 x = \lim_{x\to 0}\frac{x-\arcsin x}{\sin^3 x} \tag{2}$$

L'Hopital:

$$\lim_{x\to 0}\frac{(x-\arcsin x)'''}{(\sin^3 x)'''} =? \tag{3}$$

3 p250, #10(z), §3 Simpler

Since $\sin x \approx x$ for small x, $\sin^3 x \approx x^3$. Also $\arcsin x \approx x + \frac{1}{6}x^3$. So

$$\lim_{x\to 0}\frac{x-\arcsin x}{\sin^3 x} \approx \frac{-\frac{1}{6}x^3}{x^3} = -\frac{1}{6}$$