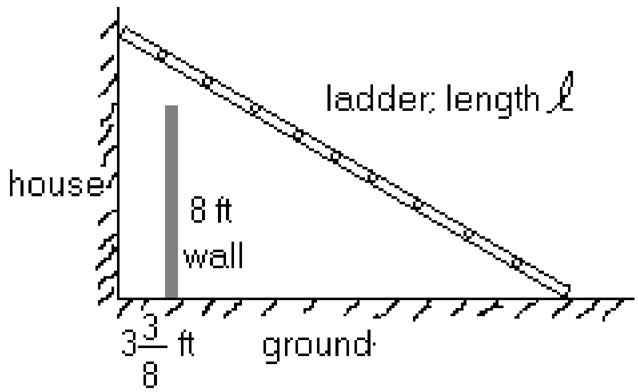
Page 127, #30

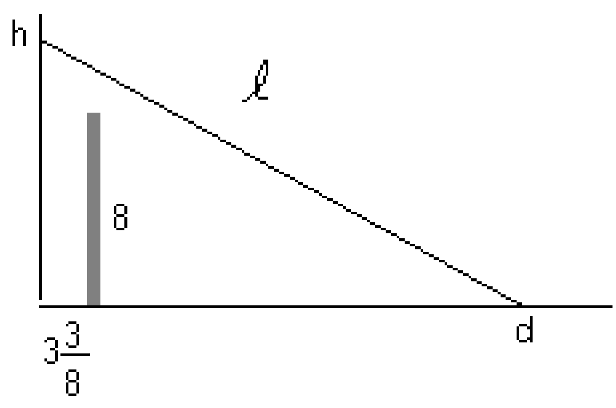
1 p127, #30, §1 Asked



Given: A free standing wall, located $3\frac{3}{8}$ ft from the side of a house.

Asked: What is the length ℓ of the shortest ladder that can reach the house (over the free standing wall).

2 p127, #30, §2 Definition

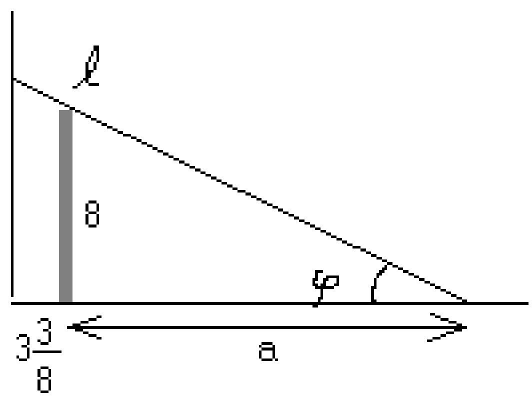


Two degrees of freedom: say h and d

One *inequality* constraint: the ladder must be above the free standing wall.

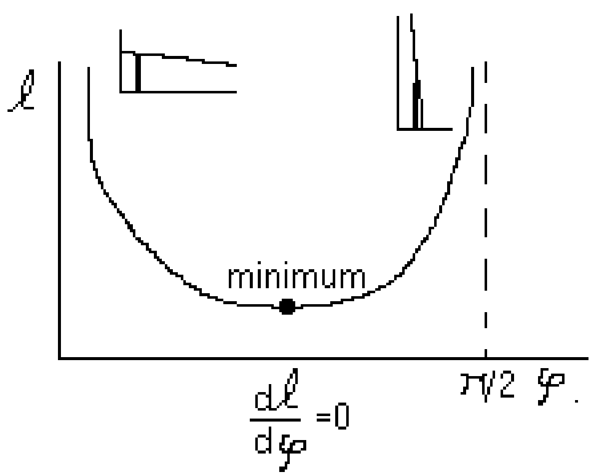
3 p127, #30, §3 Reduction

The shortest ladder hits the free standing wall:



One degree of freedom left: φ.

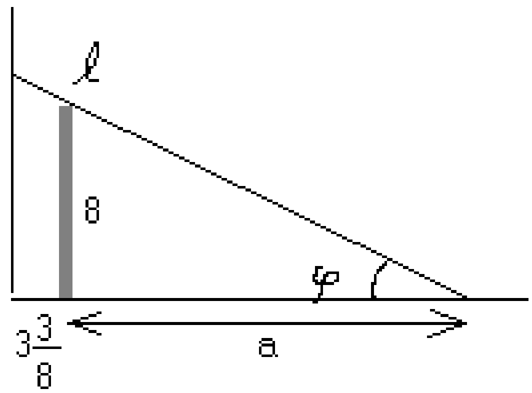
4 p127, #30, §4 Further Reduction



At the minimum:

$$\frac{d\ell}{d\varphi} = 0 \tag{1}$$

5 p127, #30, §5 Finding l



First find a:

$$a = \frac{8}{\tan\varphi}. \tag{2}$$

Then:

$$\ell = \frac{3\frac{3}{8} + a}{\cos\varphi} = \frac{3\frac{3}{8}}{\cos\varphi} + \frac{8}{\sin\varphi} \tag{3}$$

6 p127, #30, §6 Solving l'=0

$$\frac{\mathrm{d}\ell}{\mathrm{d}\varphi} = \frac{3\frac{3}{8}}{\cos^2\varphi}\sin\varphi - \frac{8}{\sin^2\varphi}\cos\varphi = 0. \tag{4}$$

$$\frac{27}{8\cos^2\varphi}\sin\varphi = \frac{8}{\sin^2\varphi}\cos\varphi \tag{5}$$

$$\tan^3\varphi = \frac{64}{27} \qquad \Longrightarrow \qquad \varphi_{\mathrm{min}} = 0.9273 \text{ radians} \tag{6}$$

7 p127, #30, §7 Finding l

From (3)

$$\ell_{\mathrm{min}} = 15.625 \text{ ft} \tag{7}$$