8.41(a)

1 8.41(a), §1 Asked

Asked:

$$\begin{vmatrix} 1 & 2 & 2 & 3 \\ 1 & 0 & -2 & 0 \\ 3 & -1 & 1 & -2 \\ 4 & -3 & 0 & 2 \end{vmatrix}$$

2 8.41(a), §2 Direct

Put in a checkerboard sign pattern (starting with +):

$$|A| = \begin{vmatrix} 1^+ & 2^- & 2^+ & 3^- \\ 1^- & 0^+ & -2^- & 0^+ \\ 3^+ & -1^- & 1^+ & -2^- \\ 4^- & -3^+ & 0^- & 2^+ \end{vmatrix}$$

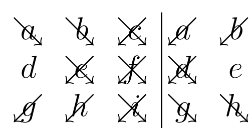
Select a row (or a column) and expand in signs, coefficients, and minors. Here the second row may be best:

$$|A| = -(1)\begin{vmatrix} 2 & 2 & 3 \\ -1 & 1 & -2 \\ -3 & 0 & 2 \end{vmatrix} - (-2)\begin{vmatrix} 1 & 2 & 3 \\ 3 & -1 & -2 \\ 4 & -3 & 2 \end{vmatrix}$$

Repeat for each of the smaller determinants until the determinants are small enough to be directly written out, eg,

$$|a| = a \qquad \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = aei + bfg + cdh - ceg - afh - bdi$$



3 8.41(a), §3 Elimination

$$|A| = \begin{vmatrix} 1 & 2 & 2 & 3 \\ 1 & 0 & -2 & 0 \\ 3 & -1 & 1 & -2 \\ 4 & -3 & 0 & 2 \end{vmatrix} \qquad \begin{matrix} (1) \\ (2) \\ (3) \\ (4) \end{matrix}$$

Interchange rows:

$$-|A| = \begin{vmatrix} 1 & 0 & -2 & 0 \\ 1 & 2 & 2 & 3 \\ 3 & -1 & 1 & -2 \\ 4 & -3 & 0 & 2 \end{vmatrix} \qquad \begin{matrix} (1') = (2) \\ (2') = (1) \\ (3) \\ (4) \end{matrix}$$

Substract multiples of the first equation from the rest:

$$-|A| = \begin{vmatrix} 1 & 0 & -2 & 0 \\ 0 & 2 & 4 & 3 \\ 0 & -1 & 7 & -2 \\ 0 & -3 & 8 & 2 \end{vmatrix} \qquad \begin{matrix} (1') \\ (2'') = (2') - (1') \\ (3') = (3) - 3(1') \\ (4') = (4) - 4(1') \end{matrix}$$

Interchange the second and third equations:

$$|A| = \begin{vmatrix} 1 & 0 & -2 & 0 \\ 0 & -1 & 7 & -2 \\ 0 & 2 & 4 & 3 \\ 0 & -3 & 8 & 2 \end{vmatrix} \qquad \begin{matrix} (1') \\ (2''') = (3') \\ (3'') = (2'') \\ (4') = (4) - 4(1') \end{matrix}$$

Substract multiples of the second equation from the rest:

$$|A| = \begin{vmatrix} 1 & 0 & -2 & 0 \\ 0 & -1 & 7 & -2 \\ 0 & 0 & 18 & -1 \\ 0 & 0 & -13 & 8 \end{vmatrix} \qquad \begin{matrix} (1') \\ (2''') \\ (3''') = (3'') + 2(2''') \\ (4'') = (4') - 3(2''') \end{matrix}$$

Replace the fourth equation by a combination of the fourth and third:

$$18|A| = \begin{vmatrix} 1 & 0 & -2 & 0 \\ 0 & -1 & 7 & -2 \\ 0 & 0 & 18 & -1 \\ 0 & 0 & 0 & 131 \end{vmatrix} \qquad \begin{matrix} (1') \\ (2''') \\ (3''') \\ (4''') = 18(4'') + 13(3''') \end{matrix}$$

The determinant of a *triangular* matrix is the product of the elements on the main diagonal:

$$18|A| = (1)(-1)(18)(131) \quad \Longrightarrow \quad |A| = -131$$