Quadratic Forms

Examples:

- quadratic curves (circles, ellipses, hyperbolae, parabolae) and surfaces (spheres, spheroids, ellipsoids, cones, cylinders, ...);

- kinetic energy of solid bodies;

- potential energy near equilibria;

- ...

Matrix form:

$$
\begin{aligned}
\vec{x}^T A \vec{x} \; = \; & a_{11}x_1^2 + a_{12}x_1x_2 + a_{13}x_1x_3 + \ldots \\
+ \; & a_{21}x_2x_1 + a_{22}x_2^2 + a_{23}x_2x_3 + \ldots \\
+ \; & \ldots
\end{aligned}
$$

An orthonormal transformation leaves the quadratic form unchanged

$$
\vec{x}'^T A' \vec{x}' = \vec{x}^T P^{T^T} P^T A P \, P^T \vec{x} = \vec{x}^T A \vec{x}
$$

but can simplify the coefficients. On principal axes

$$
\vec{x}'^T A' \vec{x}' = a_{11}' x_1'^{\,2} + a_{22}' x_2'^{\,2} + \ldots
$$