# Introduction

The *span* of a set of independent basis vectors $\vec{a}_1, \vec{a}_2, \ldots, \vec{a}_n$ is the set of all vectors $\vec{v}$ that can be described as linear combinations of the basis vectors:

$$\vec{v} = c_1\vec{a}_1 + c_2\vec{a}_2 + \ldots c_n\vec{a}_n$$

- One basis vector spans a line:
  $$\vec{v} = c_1\vec{a}_1$$
  is a 1D straight line through the origin.
- Two basis vectors span a plane:
  $$\vec{v} = c_1\vec{a}_1 + c_2\vec{a}_2$$
  is a 2D plane through the origin.
- Three basis vectors span a 3D space:
  $$\vec{v} = c_1\vec{a}_1 + c_2\vec{a}_2 + c_3\vec{a}_3$$
  is a 3D space through the origin (in $n$-dimensional space.)

Remember the definition of independence: the basis vectors are independent if the only way to get zero is to take every $c_i$ zero. It implies that you cannot express one basis vector in terms of the rest.

- Two vectors are linearly independent if they are not along the same line.
- Three vectors are linearly independent if they are not in the same plane.

The *rank* of a matrix is the number of independent rows, or columns. These are the same; see the example.