

# 3.54

## 1 3.54(a), §1 Asked

Asked: Solve:

$$\begin{array}{rcl} \left( \begin{array}{cc|c} 1 & -2 & 5 \\ 2 & 3 & 3 \\ 3 & 2 & 7 \end{array} \right) & & \begin{array}{l} (1) \\ (2) \\ (3) \end{array} \end{array}$$

## 2 3.54(a), §2 Solution

$$\begin{array}{rcl} \left( \begin{array}{cc|c} \boxed{1} & -2 & 5 \\ 2 & 3 & 3 \\ 3 & 2 & 7 \end{array} \right) & & \begin{array}{l} (1) \\ (2) \\ (3) \end{array} \end{array}$$

*Forward elimination:*

$$\begin{array}{rcl} \left( \begin{array}{cc|c} \boxed{1} & -2 & 5 \\ 0 & \boxed{7} & -7 \\ 0 & 8 & -8 \end{array} \right) & & \begin{array}{l} (1) \\ (2') = (2) - 2(1) \\ (3') = (3) - 3(1) \end{array} \end{array}$$

$$\begin{array}{rcl} \left( \begin{array}{cc|c} \boxed{2} & 3 & 3 \\ 0 & \boxed{-7} & 7 \\ 0 & 0 & 0 \end{array} \right) & & \begin{array}{l} (1) \\ (2') \\ (3'') = 7(3') - 8(2') \end{array} \end{array}$$

Echelon form. *You must bring it completely to this form.*

*Back substitution:*

From (3''), nothing; from (2'),  $y = -1$ ; from (1),  $x = 3$ .

A unique solution.

### 3 3.54(b), §3 Asked

Asked: Solve:

$$\begin{array}{l} \left( \begin{array}{cccc|c} 1 & 2 & -3 & 2 & 2 \end{array} \right) \quad (1) \\ \left( \begin{array}{cccc|c} 2 & 5 & -8 & 6 & 5 \end{array} \right) \quad (2) \\ \left( \begin{array}{cccc|c} 3 & 4 & -5 & 2 & 4 \end{array} \right) \quad (3) \end{array}$$

### 4 3.54(b), §4 Solution

$$\begin{array}{l} \left( \begin{array}{cccc|c} \boxed{1} & 2 & -3 & 2 & 2 \end{array} \right) \quad (1) \\ \left( \begin{array}{cccc|c} 2 & 5 & -8 & 6 & 5 \end{array} \right) \quad (2) \\ \left( \begin{array}{cccc|c} 3 & 4 & -5 & 2 & 4 \end{array} \right) \quad (3) \end{array}$$

Forward elimination:

$$\begin{array}{l} \left( \begin{array}{cccc|c} \boxed{1} & 2 & -3 & 2 & 2 \\ 0 & \boxed{1} & -2 & 2 & 1 \\ 0 & -2 & 4 & -4 & -2 \end{array} \right) \quad \begin{array}{l} (1) \\ (2') = (2) - 2(1) \\ (3') = (3) - 3(1) \end{array} \end{array}$$

$$\begin{array}{l} \left( \begin{array}{cccc|c} \boxed{1} & 2 & -3 & 2 & 2 \\ 0 & \boxed{1} & -2 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \quad \begin{array}{l} (1) \\ (2') \\ (3'') = (3') + 2(2') \end{array} \end{array}$$

Echelon form. *You must bring it completely to this form.*

Back substitution:

From (3''), nothing; from (2'),  $y = 1 + 2z - 2t$ ; from (1),  $x = 2 - 2(1 + 2z - 2t) + 3z - 2t = -z + 2t$ .

Solution space is 2D.

### 5 3.54(c), §5 Asked

Asked: Solve (corrected):

$$\begin{array}{l} \left( \begin{array}{cccc|c} 1 & 2 & 4 & -5 & 3 \\ 3 & -1 & 5 & 2 & 4 \\ 5 & -4 & 6 & 9 & 2 \end{array} \right) \quad \begin{array}{l} (1) \\ (2) \\ (3) \end{array} \end{array}$$

## 6 3.54(c), §6 Solution

$$\begin{pmatrix} \boxed{1} & 2 & 4 & -5 & | & 3 \\ 3 & -1 & 5 & 2 & | & 4 \\ 5 & -4 & 6 & 9 & | & 2 \end{pmatrix} \begin{array}{l} (1) \\ (2) \\ (3) \end{array}$$

*Forward elimination:*

$$\begin{pmatrix} \boxed{1} & 2 & 4 & -5 & | & 3 \\ 0 & \boxed{-7} & -7 & 17 & | & -5 \\ 0 & -14 & -14 & 34 & | & -13 \end{pmatrix} \begin{array}{l} (1) \\ (2') = (2) - 3(1) \\ (3') = (3) - 5(1) \end{array}$$

$$\begin{pmatrix} \boxed{1} & 2 & -1 & 3 & | & 3 \\ 0 & \boxed{-7} & -7 & 17 & | & -5 \\ 0 & 0 & 0 & 0 & | & -3 \end{pmatrix} \begin{array}{l} (1) \\ (2') \\ (3'') = (3') - 2(2') \end{array}$$

Echelon form. *You must bring it completely to this form.*

*Back substitution:*

Equation (3'') cannot be satisfied: there is no solution.