1.53

1 3.60(a), §1 Asked

Asked: The dimension and a basis for the solution space of the following homogeneous system:

$$\left(\begin{array}{ccccc|c} 1 & 3 & 2 & -1 & -1 & 0 \\ 2 & 6 & 5 & 1 & -1 & 0 \\ 5 & 15 & 12 & 1 & -3 & 0 \end{array}\right) \qquad \begin{array}{l} (1) \\ (2) \\ (3) \end{array}$$

2 3.60(a), §2 Solution

We need the null space of the matrix:

$$\begin{pmatrix} \boxed{1} & 3 & 2 & -1 & -1 \\ 2 & 6 & 5 & 1 & -1 \\ 5 & 15 & 12 & 1 & -3 \end{pmatrix} \qquad \begin{array}{l} (1) \\ (2) \\ (3) \end{array}$$

Forward elimination:

$$\begin{pmatrix} \boxed{1} & 3 & 2 & -1 & -1 \\ 0 & 0 & \boxed{1} & 3 & 1 \\ 0 & 0 & 2 & 6 & 2 \end{pmatrix} \qquad \begin{array}{l} (1) \\ (2') = (2) - 2(1) \\ (3') = (3) - 5(1) \end{array}$$

$$\begin{pmatrix} \boxed{1} & 3 & 2 & -1 & -1 \\ 0 & 0 & \boxed{1} & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad \begin{array}{l} (1) \\ (2') \\ (3'') = (3') - 2(2') \end{array}$$

Continue to row canonical:

$$\begin{pmatrix} \boxed{1} & 3 & 0 & -7 & -3 \\ 0 & 0 & \boxed{1} & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad \begin{array}{l} (1') = (1) - 2(2') \\ (2') \\ (3'') \end{array}$$

Back substitution:

From $(3'')$, nothing; from $(2')$, $z = -3s - t$; from (1'), $x = -3y + 7s + 3t$. Variables y, s, and t cannot be determined: space is 3D.

Vector form;

$$\begin{pmatrix} x \\ y \\ z \\ s \\ t \end{pmatrix} = y \begin{pmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + s \begin{pmatrix} 7 \\ 0 \\ -3 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 3 \\ 0 \\ -1 \\ 0 \\ 1 \end{pmatrix}$$

The three vectors in the right hand side form a basis for the solution space.