Bases

A basis $\vec{a}_1, \vec{a}_2, \ldots, \vec{a}_n$ to a space is a chosen set of vectors so that any vector $\vec{x}$ in the space can be *uniquely* expressed in terms of the basis vectors:

$$\vec{x} = c_1\vec{a}_1 + c_2\vec{a}_2 + \ldots + c_n\vec{a}_n$$

Example: $\hat{\imath}, \hat{\jmath}, \hat{k}$ are a basis to coordinate space:

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$$

Basis vectors must be *independent*, which means you need all of them to express any arbitrary vector in the space. If some basis vector can be expressed in terms of the others, you do not need that vector and should throw it out.

For example if $\hat{\imath}$ would be $\hat{\jmath} + \hat{k}$, then you would not need it, since $x\hat{\imath}$ could then be written as $x\hat{\jmath} + x\hat{k}$. But there is no way to get $\hat{\imath}$ from a linear combination of $\hat{\jmath}$ and $\hat{k}$

To check independence of supposed basis vector $\vec{a}_1, \vec{a}_2, \ldots, \vec{a}_n$, verify that

$$c_1\vec{a}_1 + c_2\vec{a}_2 + \ldots + c_n\vec{a}_n = 0$$

only when all coefficients $c_1, c_2, \ldots, c_n$ are zero.

Why this works: If, for example, c_1 would be nonzero, you can take $c_1\vec{a}_1$ to the other side and divide by $-c_1$.