1.41

1 1.41, §1 Asked

Given: The vectors

$$\vec{u} = (1, -2, 4) \qquad \vec{v} = (3, 5, 1)$$

Asked: various.

2 1.41, §2 Solution

$$\vec{u} = (1, -2, 4) \qquad \vec{v} = (3, 5, 1)$$

Sum:

$$3\vec{u} - 2\vec{v} = (3\ 1 - 2\ 3, 3\ (-2) - 2\ 5, 3\ 4 - 2\ 1) = (-3, -16, 10)$$

Dot product:

$$\vec{u} \cdot \vec{v} = 1\ 3 + (-2)\ 5 + 4\ 1 = -3$$

Norm or length:

$$||\vec{u}|| = \sqrt{1^2 + (-2)^2 + 4^2} = \sqrt{21} \qquad ||\vec{v}|| = \sqrt{3^2 + 5^2 + 1^2} = \sqrt{35}$$

Angle between vectors:

$$\cos(\vartheta) = \vec{u} \cdot \vec{v}/(||\vec{u}||\ ||\vec{v}||) = -3/\sqrt{21}\sqrt{35} = -3/(7\sqrt{15})$$

Distance between the end points:

$$d(\vec{u}, \vec{v}) = ||\vec{u} - \vec{v}|| = ||(-2, -7, 3)|| = \sqrt{4 + 49 + 9} = \sqrt{62}$$

Projection:

$$\text{proj}(\vec{u}, \vec{v}) = (\vec{u} \cdot \vec{v}/||\vec{v}||)(\vec{v}/||\vec{v}||) = (-3/35)(3, 5, 1) = (-9/35, -3/7, -3/35)$$