1.54

1   1.54, §1 Asked

Given: The vectors
$$\vec{u} = 3\hat{\imath} - 4\hat{\jmath} + 2\hat{k} \qquad \vec{v} = 2\hat{\imath} + 5\hat{\jmath} - 3\hat{k}$$

Asked: Various

2   1.54, §2 Solution

Given:
$$\vec{u} = 3\hat{\imath} - 4\hat{\jmath} + 2\hat{k} \qquad \vec{v} = 2\hat{\imath} + 5\hat{\jmath} - 3\hat{k}$$

Sum:
$$2\vec{u} - 3\vec{v} = 6\hat{\imath} - 8\hat{\jmath} + 4\hat{k} - 6\hat{\imath} - 15\hat{\jmath} + 9\hat{k} = -23\hat{\jmath} + 13\hat{k}$$

Dot product:
$$\vec{u} \cdot \vec{v} = (3\hat{\imath} - 4\hat{\jmath} + 2\hat{k}) \cdot (2\hat{\imath} + 5\hat{\jmath} - 3\hat{k})$$
Use the fact that $\hat{\imath} \cdot \hat{\imath} = \hat{\jmath} \cdot \hat{\jmath} = \hat{k} \cdot \hat{k} = 1$ and $\hat{\imath} \cdot \hat{\jmath} = \hat{\jmath} \cdot \hat{k} = \hat{k} \cdot \hat{\imath} = 0$:

$$\vec{u} \cdot \vec{v} = 3\ 2 + 0 + 0 - 0 - 4\ 5 + 0 - 0 + 0 - 2\ 3 = -20$$

Norm or length:
$$||\vec{u}|| = \sqrt{\vec{u} \cdot \vec{u}} = \sqrt{(3\hat{\imath} - 4\hat{\jmath} + 2\hat{k}) \cdot (3\hat{\imath} - 4\hat{\jmath} + 2\hat{k})}$$
Multiply out as before:
$$||\vec{u}|| = \sqrt{3^2 + (-4)^2 + 2^2} = \sqrt{29}$$