1.59(b)

1 1.59(b), §1 Asked

Given: The hyperboloid of one sheet

$$x^2 + 3y^2 - 5z^2 = 160$$

and the point P with position vector (3,-2,1) on that hyperboloid.

Asked: A normal vector $\vec{N}$ to the surface at P and the tangent plane at P.

2 1.59(b), §2 Solution

$$x^2 + 3y^2 - 5z^2 = 160 \qquad P = (3, -2, 1)$$

Correct problem:

$$x^2 + 3y^2 - 5z^2 = 16 \qquad P = (3, -2, 1)$$

Bring equation of surface in *standard form* (zero right hand side):

$$x^2 + 3y^2 - 5z^2 - 16 \equiv F(x, y, z) = 0$$

A normal vector to a surface in standard form is given by the gradient of F:

$$\nabla F \equiv \begin{pmatrix} \frac{\partial F}{\partial x} \\ \frac{\partial F}{\partial y} \\ \frac{\partial F}{\partial z} \end{pmatrix} = \begin{pmatrix} 2x \\ 6y \\ -10z \end{pmatrix}$$

At P, $(x, y, z) = (3, -2, 1)$, so:

$$\vec{N} = \nabla F|_P = \begin{pmatrix} 6 \\ -12 \\ -10 \end{pmatrix}$$

Tangent plane:

$$\vec{N} \cdot \vec{r} = \vec{N} \cdot \vec{r}_P$$

or

$$6x - 12y - 10z = 6\,3 - 12\,(-2) - 10\,1 = 32$$

Can divide by 2 to simplify:

$$3x - 6y - 5z = 16$$