5.38

1 5.38, §1 Asked

Solve:

$$xy' + y = xy^3$$

2 5.38, §2 Solution

$$xy' + y = xy^3$$

It is a Bernoulli equation since it has terms linear in y and a power of y.

$$xy^{-3}y' + y^{-2} = x$$

Put $u = y^{-2}$:

$$-\tfrac{1}{2}xu' + u = x$$

Solution of the homogeneous equation:

$$-\tfrac{1}{2}xu' + u = 0 \quad \Longrightarrow \quad \frac{\mathrm{d}u}{u} = 2\frac{\mathrm{d}x}{x} \quad \Longrightarrow \quad u = Cx^2$$

Solution of the inhomogeneous equation:

$$u = C(x)x^2$$

into the inhomogeneous equation:

$$-\tfrac{1}{2}xC'x^2 - \tfrac{1}{2}xC2x + Cx^2 = x$$

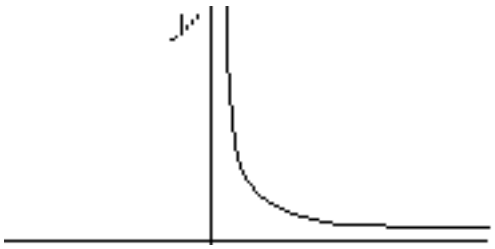
$$C' = -\frac{2}{x^2} \quad \Longrightarrow \quad C = \frac{2}{x} + C_0$$

$$u = C(x)x^2 = 2x + C_0x^2 = \frac{1}{y^2}$$

Solution:

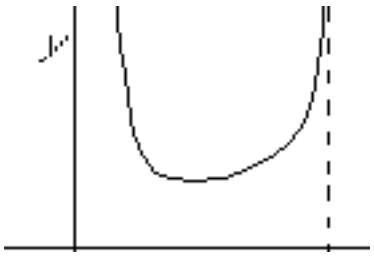
$$y = \frac{\pm 1}{\sqrt{2x + C_0x^2}}$$

For $C_0 = 0$ $y = \pm 1/\sqrt{2x}$:

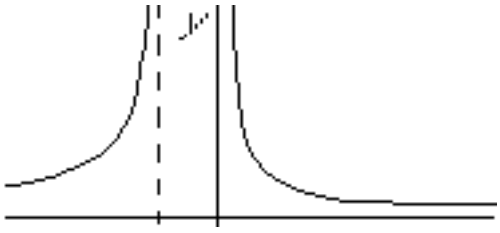


For $x = -2/C_0$, y is infinite.

For $C_0 < 0$:



For $C_0 > 0$:



Total: