Exact Equations

If for an equation of the form

$$g_1(x, y)\,\mathrm{d}x + g_2(x, y)\,\mathrm{d}y = 0,$$

the cross derivatives of the coefficients are equal;

$$\frac{\partial g_1}{\partial y} = \frac{\partial g_2}{\partial x},$$

then the equation is exact.

The solution of an exact equation is:

$$g(x, y) = C$$

where $g(x, y)$ is found by solving

$$\frac{\partial g}{\partial x} = g_1(x, y) \qquad \frac{\partial g}{\partial y} = g_2(x, y).$$

You do that by first solving the easier of the two, giving an integration constant that depends on the other variable. For example, solving $\partial g/\partial x = g_1(x, y)$ gives an integration constant depend on y. Next you take that solution and put it into the other equation.

If an equation is not exact, you may sometimes be able to find an "integrating factor" in a table.