First Order Systems

Important for numerical work. Library subroutines usually do not solve higher order equations, but they do solve first order systems.

General First Order System:

$$\vec{y}' = \vec{f}(x, \vec{y})$$

Written out

$$\begin{aligned} y_1' &= f_1(x, y_1, y_2, \ldots, y_n) \\ y_2' &= f_2(x, y_1, y_2, \ldots, y_n) \\ &\cdots \\ y_n' &= f_n(x, y_1, y_2, \ldots, y_n) \end{aligned}$$

If the functions are linear constant coefficient ones, we can rewrite this as:

$$\vec{y}' = A\vec{y} + b(x).$$

In this class, solution using eigenvalues and eigenvectors is *required.* We assume that A is diagonalizable.

Homogeneous solution:

$$y_h = C_1\vec{v}_1 e^{\lambda_1 x} + C_2\vec{v}_2 e^{\lambda_2 x} + \ldots$$

where $\lambda_1, \lambda_2, \ldots$ are the eigenvalues of A and $\vec{v}_1, \vec{v}_2, \ldots$ the eigenvectors.

General solution: Guess and add a particular solution. Varying the parameters C_1, C_2, ... also works.