6.34

1 6.34, §1 Asked

Solve:

$$y' + x^2 y = x^2$$

2 6.34, §2 Solution

$$y' + x^2 y = x^2$$

The equation is linear.

Solution of the homogeneous equation:

$$y' + x^2 y = 0 \quad \Longrightarrow \quad \frac{\mathrm{d}y}{y} = -x^2 \,\mathrm{d}x$$

$$\ln|y| = -\tfrac{1}{3}x^3 + C_1 \quad \Longrightarrow \quad y = Ce^{-\frac{1}{3}x^3}$$

Solution of the inhomogeneous equation:

$$y = C(x)e^{-\frac{1}{3}x^3}$$

into

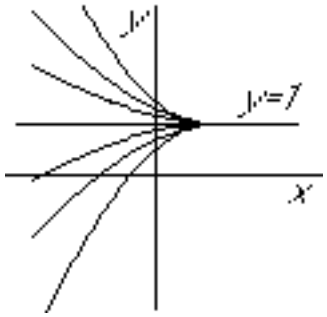
$$y' + x^2 y = x^2$$
$$C'e^{-\frac{1}{3}x^3} - Ce^{-\frac{1}{3}x^3}x^2 + x^2 Ce^{-\frac{1}{3}x^3} = x^2$$
$$C' = x^2 e^{\frac{1}{3}x^3} \quad \Longrightarrow \quad C = e^{\frac{1}{3}x^3} + C_0$$

Solution:

$$y = C(x)e^{-\frac{1}{3}x^3} = 1 + C_0 e^{-\frac{1}{3}x^3}$$

Note: function $y(x) = 1$ is called a particular solution. It is *one* solution that satisfies the inhomogeneous equation.

The general solution of linear equations is always: (any arbitrary particular solution) plus (the general solution of the homogeneous equation).



(What is wrong in the graph above)?