11.45

1 11.45, §1 Asked

Solve:

$$y'' - 2y' + y = 3e^{2x}$$

2 11.45, §2 Solution

$$y'' - 2y' + y = 3e^{2x}$$

Homogeneous equation:

Characteristic polynomial:

$$\lambda^2 - 2\lambda + 1 = 0$$

has roots $\lambda_1 = \lambda_2 = 1$: General solution:

$$y_h = C_1 e^x + C_2 x e^x$$

Particular solution:

$$y_p'' - 2y_p' + y_p = 3e^{2x}$$

Guessing $y_p = Ce^{2x}$ produces

$$y_p'' - 2y_p' + y_p = C\left(4e^{2x} - 4e^{2x} + e^{2x}\right) = Ce^{2x}$$

so $C = 3$ and $y_p = 3e^{2x}$.

Total solution:

$$y = 3e^{2x} + C_1 e^x + C_2 x e^x$$