

12.10

1 12.10, §1 Asked

Solve:

$$y'' + y = \sec x$$

2 12.10, §2 Solution

$$y'' + y = \sec x$$

Homogeneous equation:

$$\lambda^2 + 1 = 0 \implies \lambda = \pm i$$

$$y_h = A \cos x + B \sin x$$

Variation of parameters:

$$y = A \cos x + B \sin x \quad A = A(x), B = B(x)$$

$$y' = -A \sin x + B \cos x + A' \cos x + B' \sin x$$

Put the additional terms to zero:

$$A' \cos x + B' \sin x = 0 \tag{1}$$

$$y'' = -A \cos x - B \sin x - A' \sin x + B' \cos x$$

Do not put the additional terms to zero in the highest derivative. Instead, put everything into the O.D.E.:

$$\begin{aligned} y'' + y &= -A \cos x - B \sin x - A' \sin x + B' \cos x + A \cos x + B \sin x = \sec x \\ -A' \sin x + B' \cos x &= \sec x \end{aligned} \tag{2}$$

The result is a system of linear equations (1), (2) for A' and B' :

$$\begin{pmatrix} \cos x & \sin x & \left| \begin{array}{c} 0 \\ \sec x \end{array} \right. \end{pmatrix} \quad \begin{matrix} (1) \\ (2) \end{matrix}$$

Forward elimination:

$$\begin{pmatrix} \cos x & \sin x & \left| \begin{array}{c} 0 \\ 1 \end{array} \right. \end{pmatrix} \quad \begin{matrix} (1) \\ (2') = \cos x(2) + \sin x(1) \end{matrix}$$

Back substitution gives $B' = 1$ and $A' = -\tan x$:

$$B = x + B_0 \quad A = \ln |\cos x| + A_0$$

Total solution $y = A \cos x + B \sin x$:

$$y = \ln |\cos x| \cos x + x \sin x + A_0 \cos x + B_0 \sin x$$