EML 5060 | **Analysis in Mechanical Engineering** | **09/28/18**
Closed book | *Van Dommelen* | *12:30-1:20 pm*

Solutions should be fully *derived* showing all intermediate results, using class procedures. *Show all reasoning.* Bare answers are absolutely not acceptable, because I will assume they come from your calculator (or the math handbook, sometimes,) instead of from you. You must state what result answers what part of the question. Answer what is asked; you do not get any credit for making up your own questions and answering those. Use the stated procedures. Give exact, fully simplified, answers where possible.

One book of mathematical tables, such as Schaum's Mathematical Handbook, may be used, as well as a calculator, and a handwritten letter-size formula sheet.

1. **Background:** Graphical depiction of a function is often an essential part to understand its properties.

   Question: Analyze and very neatly graph

   $$y = \sqrt[3]{x^3 + 3x^2} \qquad \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \frac{-2x^2}{(x^3 + 3x^2)^{5/3}}$$

   Discuss x and y intercepts and extents, asymptotic behavior for large positive x and large negative x, horizontal, oblique and vertical asymptotes, symmetries, local and global maxima and minima, concavity, inflection points, kinks, cusps, vertical slopes and other singularities.

   Draw the function very neatly and precisely, on suitably labelled axes, clearly showing all features.

2. **Background:** One thing virtually any engineer, regardless of type of job, must be able to do is estimate errors in results from estimated errors in data.

   Question: Going down in water, the pressure increases as

   $$P = P_{\text{atm}} - \rho_{\text{water}} g h$$

   where P_{atm} is the atmospheric pressure at the considered location, ρ_{water} the density of, you guessed it, g the acceleration of gravity at the location, and $h \le 0$ the height above the free surface of the water. Given that at the considered location

   $$P_{\text{atm}} = 100{,}000 \text{ Pa} \pm 0.7\% \qquad \rho_{\text{water}} = 997 \text{ kg/m}^3 \pm 0.1\% \qquad g = 9.8 \text{ m/s}^2 \pm 0.2\%$$

   what is the maximum relative error in P at height $h = -20$?

3. **Background:** Areas of plates are important for material cost, weight, dynamics, stresses, etcetera.

   Question: Consider the region

   $$\text{inside} \quad x^2 + y^2 = 2x \quad \text{and} \quad x \ge 1$$

   Derive the area of the region above by using multiple integration in polar coordinates r and θ. In particular, write the complete integrals to do if you do r first, and if you do θ first. Give a complete description of all the disadvantages that each of the two ways of integration has. (Coordinates x, y, r, and θ are the normal coordinates, and may not be modified. Nor may you modify the limits of integration by only integrating half of it and multiplying by 2.)