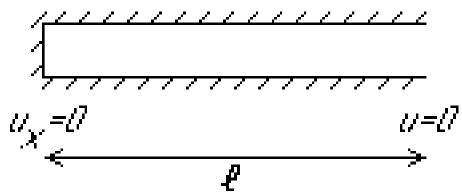
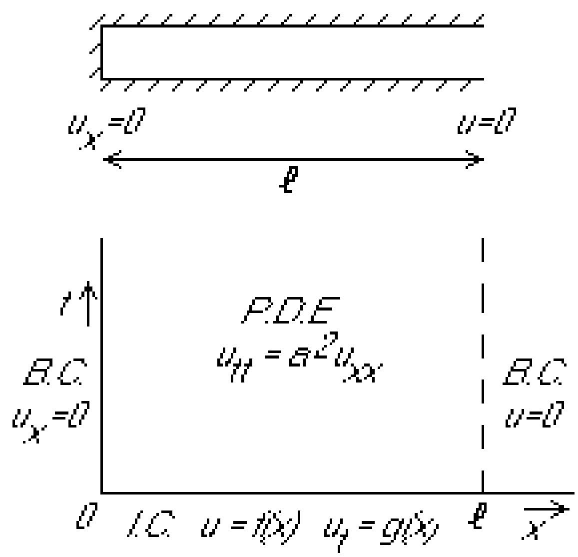
7.28

1 7.28, §1 Asked

Asked: Find the pressure for sound wave propagation in a tube with one end closed and one end open.



2 7.28, §2 P.D.E. Model



- Finite domain $\bar{\Omega}$: $0 \le x \le \ell$
- Unknown pressure $u = u(x, t)$
- Hyperbolic
- Two initial conditions

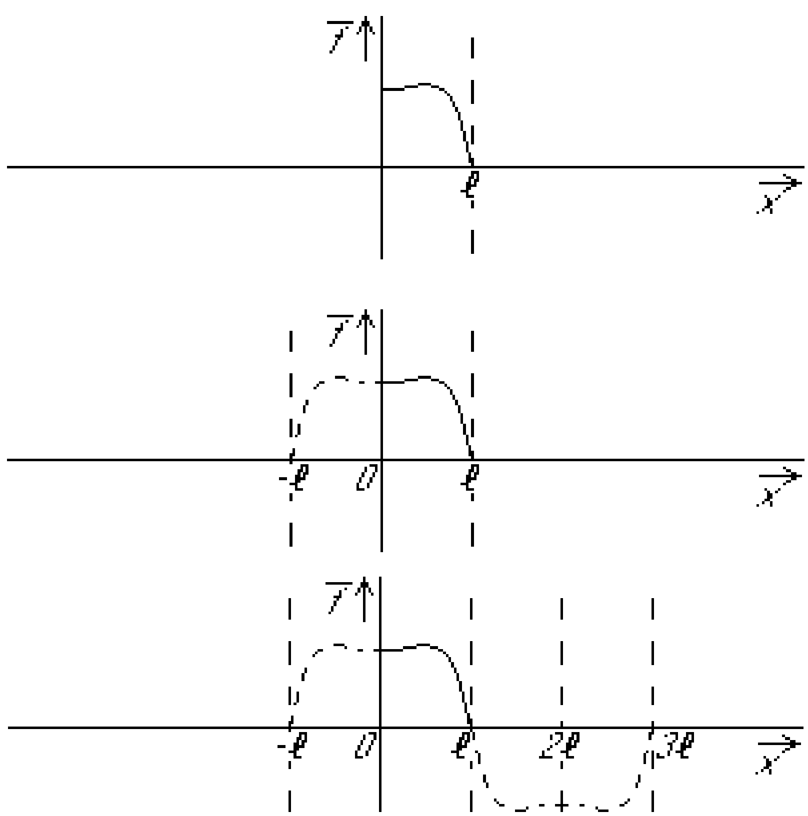
- One homogeneous Neumann boundary condition at $x = 0$ and a homogeneous Dirichlet condition at $x = \ell$.
- Constant speed of sound a and much smaller flow velocities.

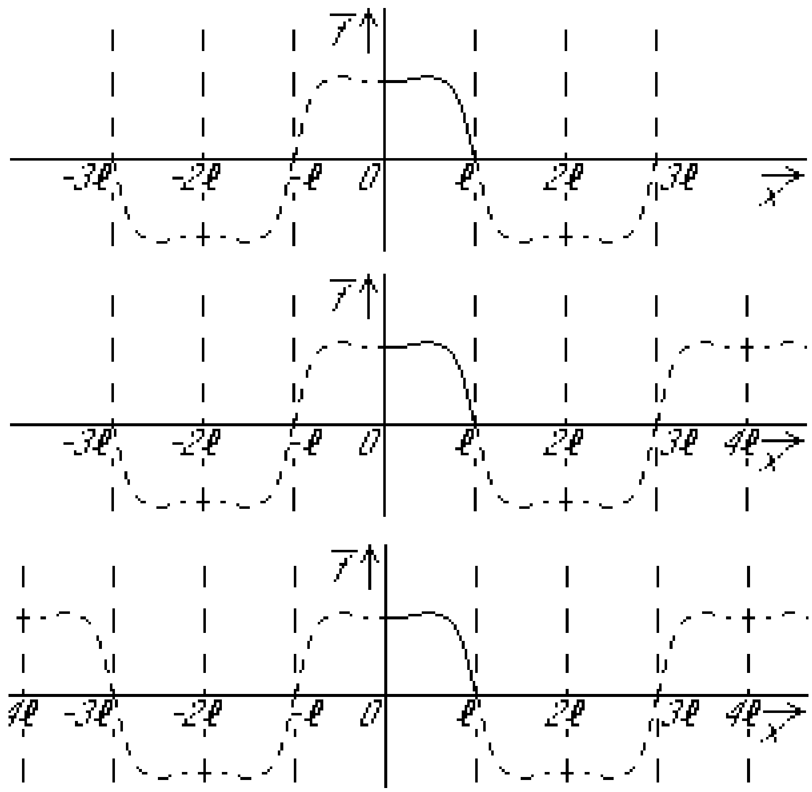
Try D'Alembert

3 7.28, §3 Boundaries

- Get rid of the boundaries by imagining that the pipe extends from $-\infty < x < \infty$
- To do so, we must extend the initial conditions $f(x)$ and $g(x)$ to all x. Call the extended functions $\bar{f}(x)$ (or F) and $\bar{g}(x)$ (or G).
- The extended functions should make the given boundary conditions automatic.

To make the boundary condition $u_x = 0$ at $x = 0$ automatic, create *symmetry* around $x = 0$ To make the boundary condition $u = 0$ at $x = \ell$ automatic, create *antisymmetry* (odd symmetry) around $x = \ell$.



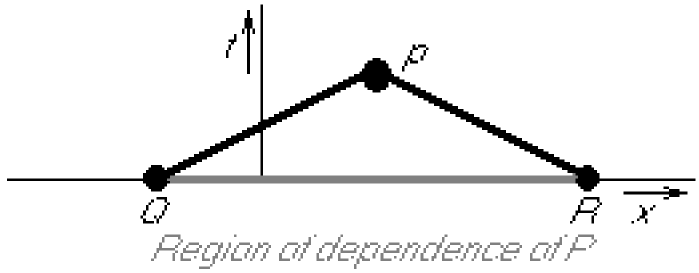


Create the extended function $\bar{g}(x)$ or G the same way.

4 7.28, §4 Solution

$$u(x,t) = \frac{\bar{f}(x-at) + \bar{f}(x+at)}{2} + \frac{1}{2a}\int_{x-at}^{x+at} \bar{g}(\xi)\,\mathrm{d}\xi$$

Probably pretty easy to evaluate.



In the range $0 \leq x \leq \ell$, the found solution is exactly the same as for the finite pipe!

Note that if f and/or g does not satisfy the given boundary conditions, $\bar{f}$ and $\bar{g}$ may have kinks or jumps.